MIMO Radar Waveform Design Criteria

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Abstract—Multi-Input Multi-Output (MIMO) radars utilize multiple transmitters and multiple receivers. The transmitted waveforms are independent of each other. This allows for multiple waveforms to choose from in order to achieve a specific purpose. However, there is possible interference which degrades the radar performance in target detection, identification and tracking. A comparison of various available design methods is done. The requirements and performance of each design method are used to propose their suitability in different applications.

Keywords—MIMO radar, target detection, waveform design, waveform optimization.

I. INTRODUCTION

The basic operation of radars involves sending out electromagnetic signals that are reflected by targets. The nature of the echo signal provides information about the target e.g. range, radial velocity, angular direction, size, shape etc [1]. This signal is referred to as the radar waveform.

Multiple-Input Multiple-Output (MIMO) radar has significantly improved the performance of radar over traditional phased array radar via integrating target scattering information from multiple diversified channels [2], [3]. According to [1], MIMO radar can exploit the spatial diversity of target scatters, and enables a variety of new techniques that can improve the performance in many aspects.

In the area of radar target identification and classification, there exists some recent work which applied both information theoretic and estimation theoretic criteria for optimal design [4]. In [1], the authors assume that the radar transmitter has knowledge of the target’s second order statistics and that the transmitted power is constrained and with these assumptions, investigate the optimal radar waveform design based on the following two criteria:

- maximizing the conditional Mutual Information (MI) between the random target impulse response and the reflected radar waveforms;
- minimizing the value of minimum mean-square error (MMSE) in estimating the target impulse response.

In [5], the problem of radar waveform design for maximizing the signal-to-noise ratio (SNR) at the output of the receiver filter is considered.

A criteria that finds signals that discriminate between a collection of targets of interest after observing the backscatter from an illuminated unknown target is investigated in [6].

According to [7], there are various types of waveform design criteria that have been employed to achieve optimal waveforms including:
1. Maximizing Mutual Information and Minimizing Mean-Square error.
2. Minimax Robust design.
4. Waveform covariance matrix design.
5. In frequency domain

This paper describes the first three which are commonly used.

The remainder of the paper is organized as follows. Section II describes the various types of design criteria that have been studied in current literature and the optimal waveforms that are generated from the design criteria. A comparison and discussion of the design methods is provided in section III and the paper is concluded in section IV.

Notation: bold upper case letters are used to denote matrices, and bold lower case letters to signify column vectors. We use det{·} and tr{·} for the determinant and trace of a matrix, \( \Re \{ \cdot \} \) for the real part of a complex number, \( \& \) for Kronecker product, diag{a} for a diagonal matrix with its diagonal given by the vector a, and \( (a)^+ \) for the nonnegative part of a real number a, i.e., \( (a)^+ = \max [0, a] \).

II. DESIGN CRITERIA

We consider here MIMO radar systems with multiple transmit and multiple receive antennas which may be distributed in space. Different criteria are presented that are used to come up with optimal transmit waveforms.

A. Maximizing Mutual Information

In 1993, Bell published his paper [5] which suggested maximizing the mutual information (MI) between the random target impulse response and the reflected radar signal to design radar waveforms. The study implied that the greater the MI between the target impulse response (target reflection) and the measurement (the reflected signals), the better the capability of the radar to estimate the parameters describing the target. In [1] the conditional MI is considered between the received waveform \( (y') \), and the target impulse response \( (g) \) given the...
knowledge of the transmitted waveform \( \overline{X} \). The MI is given by
\[
I = \log(\det(X^\mathsf{H}X + \sigma_n^2 I_{QL})) - \log(\det(\sigma_n^2 I_{QL}))
\]
(1)

It is shown that \( \overline{y} \) is Gaussian distributed with zero mean and covariance \( (X^\mathsf{H}X + \sigma_n^2 I_{QL}) \).

The MI is thus given by [1]
\[
I = \log(\det(X^\mathsf{H}X + \sigma_n^2 I_{QL})) - \log(\det(\sigma_n^2 I_{QL}))
\]
(2)

The goal is to define an optimum value of \( X \) that maximizes the MI under a constraint on the total transmit power given in [1] as \( tr\{X^\mathsf{H}X\} \leq LQP_0 \).

The problem of waveform design based on MI can then be expressed as

\[
\text{Problem 1: } \max_X \det((X^\mathsf{H}X + \sigma_n^2 I_{QL}))
\]
(3)

\[
s.t. \quad tr\{X^\mathsf{H}X\} \leq LQP_0
\]

B. Minimizing the MMSE

This is done under the scenario of target identification. It can be verified that, conditioned on \( X \), \( \overline{g} \) and \( \overline{y} \) are jointly Gaussian distributed and the conditional distribution of \( \overline{g} \) given \( \overline{y} \) and \( X \) is also Gaussian [4].

The Bayes estimate, which is an MMSE estimator [4] is given by
\[
\hat{\overline{g}} = (X^\mathsf{H}X + \sigma_n^2 \Sigma_g^{-1})^{-1} X^\mathsf{H} \overline{y}
\]
and the Bayes risk is given by
\[
\xi = E\{|| \hat{\overline{g}} - \overline{g} ||^2\}
\]
\[
= tr\{(\sigma_n^2 X^\mathsf{H}X + \Sigma_g^{-1})^{-1}\}
\]
(4)

The goal is to find \( X \) (transmitted waveforms) that minimize the value of MMSE under a constraint on the total transmit power \( tr\{X^\mathsf{H}X\} \leq LQP_0 \).

The problem of waveform design based on MMSE estimation is expressed as in [4]:

\[
\text{Problem 2: } \min_X \quad tr\{(\sigma_n^2 X^\mathsf{H}X + \Sigma_g^{-1})^{-1}\}
\]
(5)

\[
s.t. \quad tr\{X^\mathsf{H}X\} \leq LQP_0.
\]

C. Minimax Robust Design

Instead of assuming that the exact characterization of the target PSD is available, the study in [4] assumes that the actual PSD is only known to lie in some class of possible PSDs. Based on this, the authors employ a statistical theory of minimax robustness to find robust waveform designs under both the MI and the MMSE criteria.

Using a minimax robust design optimizes the worst-case performance, and is similar to designing for the worst case which is a well accepted engineering approach [4].

To loosen the requirements for a priori knowledge about the target, the research in [4] adopts a band model by assuming that the PSD lies in an uncertainty class of spectra bounded by known upper and lower bounds. The minimax robust waveform is the optimum waveform designed for the PSD upper bounds when the MMSE is the criterion of interest, and is the optimum waveform designed for the PSD lower bounds when the MI criterion is adopted. The reader is referred to [4] for details on the statistical analysis and numerical results.

D. Maximizing the SINR ratio

A waveform design is to be chosen to achieve waveforms which when optimized will give the highest signal-to-noise ratio required for a maximum number of targets that can uniquely be identified.

In [8], the author studies the generation of optimal waveforms that maximize the signal-to-interference-plus-noise-ratio (SINR) at the output of a general detector. This design approach requires prior knowledge of the target and clutter statistics.

A phase modulated baseband waveform is designed in [9], and optimized by maximizing the SINR at the receiver filter output. Here the target impulse response is modeled as
\[
h(t) = \sum_{i=1}^{k} h_i \delta(t - t_i)
\]
(7)

where \( 0 \leq t_i \leq \tau_{\max} \); \( t_i \) is the different delays of the reflection centers and \( h_i \) the reflection coefficients.

The echo signal is given by
\[
y(t) = \int_{0}^{\tau_{\max}} h(\tau)x(t - \tau)d\tau
\]
(8)

Where \( x(t) \) is the transmit signal.

The waveform design problem is further defined as follows. Given a target impulse response \( h(t) \) and a stationary additive Gaussian noise \( n(t) \) with the power spectral density \( P_{nn}(f) \), find a baseband waveform \( u(t) \) with total energy \( E \) and a receiver-filter impulse response \( r(t) \) such that the SNR of the receiver output \( q(t) \) is maximized at time \( t_o \) [9].

The output at the receiver \( q(t) \) is composed of a signal component \( q_s(t) \) and a noise component \( q_n(t) \). The signal-to-noise ratio which is the energy ratio between the signal and the noise is given by
The signal-to-noise ratio thus becomes:
\[ \text{SNR} = \frac{\int_{-B/2}^{B/2} \left| H(f + f_c)U(f)R(f)\right|^2 df}{\int_{-B/2}^{B/2} \left| R(f) \right|^2 P_{nn}(f) df} \]

(9)

IV. CONCLUSION
This paper has analyzed the various approaches to designing of optimized MIMO radar waveforms. Each design method follows a specific criterion with an aim of giving the best radar performance. A comparison of the design methods showed that each one is suitable for different applications. This information is useful in cases where improvements need to be made on various aspects of radar performance e.g. target detection, target identification, target tracking, etc. The improvement would be done based on a design criterion that is suitable for the application.

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REFERENCES