Interaction between the Linear and Nonlinear Polarization in Optical Fibre.


Abstract— Interaction between polarization mode dispersion (PMD) and nonlinear induced birefringence in a fibre with consideration of mode coupling has been investigated theoretically using Matlab and Optism. The investigations show that PMD leads to a distortion of the signal but when it is frequency independent, its overall effect is just a rotation of the signal state of polarization (SOP) on the Poincare sphere. Further investigation show that the effect of nonlinear birefringence alone depolarizes the signal, while in high PMD links where polarization mode coupling is high, the nonlinear birefringence effect couples with second-order PMD such that it may reduce the penalty and improve the signal DOP. It is also found that when nonlinear birefringence becomes significant, asymmetry arises between the two principal axes of the fibre, such that it is only one axis which experiences the effect of nonlinear birefringence.

Keywords: PMD, nonlinear birefringence, DOP, polarization mode coupling.

I. INTRODUCTION

Optical fibres as transmission medium are never perfect, they have slight asymmetries or other perturbations that destroy the degeneracy, leading to two polarization states with slightly different phase and group velocities, a phenomenon known as birefringence [1]. The birefringence in the fibre contributes to polarization mode dispersion (PMD) which is a major limiting factor that leads to system impairments for the transmission of high speed optical signals over the already embedded optical fibre network.

Nonlinear effects become significant at high optical power levels and have become more important since the development of Erbium-doped fibre amplifier (EDFA) and wavelength division multiplexed (WDM) systems. The origin of the nonlinearities is the refractive index of the optical fibre, which varies with the intensity of the optical signal. The high signal intensity associated with the large number of channels at closely spaced wavelengths may introduce problematic nonlinear effects such as four-wave mixing (FWM), self-phase modulation (SPM) and cross-phase modulation (XPM). Thus, the intensity dependence of the refractive index (optical Kerr effect) is responsible for nonlinear birefringence whose effect is nonlinear polarization rotation [2]. These effects just like the linear effects can be detrimental in optical communications, but they also have many useful applications, especially for the implementation of all-optical functionalities in optical networks [3]. It is therefore important to analyze the interaction between the linear and nonlinear effects in the transmission system in order to utilize their potential to the fullest. In this paper, investigation of the effect of birefringence and pump power on the probe signal was carried out. Further study was done on the influence of relative angles of the pump signal, nonlinear birefringence and pump optical power on the probe DOP.

II. THEORY

Single-mode optical fibres ideally are supposed to maintain a single polarization state even after long distance transmission. In practice, the optical pulse propagates along single-mode fibre in two polarization modes [4] due to asymmetry in the fibre cross-section. The consequence of this asymmetry of cross-section is the existence of optical birefringence. As a result of birefringence, a pulse launched into the fibre at a particular state of polarization, split into two identical, linearly polarized pulses having their electric field vectors aligned with the symmetry axes of the fibre and having different group velocities. The pulses arrive at the output differentially delayed as shown in Fig. 1. The difference in the transmission time of two pulses polarized along the states of polarization producing the shortest and longest propagation times is known as the Differential Group Delay (DGD) given by (1). The DGD is the measure of an effect known as polarization mode dispersion (PMD), which is a phenomenon that leads to pulse broadening and system impairments limiting the transmission capacity of the fibre [5], [6].
Fig. 1: Effect of PMD in a birefringent fibre on optical pulse.

\[
\Delta T = \frac{L}{V_{gx}} - \frac{L}{V_{gy}} = L|\beta_{1x} - \beta_{1y}| = L(\Delta \beta_1) \quad \text{..................................(1)}
\]

Where \( L \) is the length of the fibre, \( V_{gx} \) and \( V_{gy} \) are group velocities along the x and y axes of the fibre respectively and \( (\Delta \beta_1) \) is the modal birefringence.

When WDM system is considered with two channels, a pump signal at frequency \( \omega_p \) and a probe signal at frequency \( \omega_s \), then for a pump optical power much higher than that of the probe, the nonlinear effects of the probe may be neglected while maintaining those of the pump. The pump modulates the phases of both signals through the nonlinear effects, self-phase modulation and cross-phase modulation. Taking the probe as stationary, the coupled nonlinear Schrodinger equations (NLSE) can be written as

\[
\frac{\partial |A_p\rangle}{\partial z} + \frac{i}{2} \Omega \mathbf{b} \bullet \mathbf{\sigma} |A_s\rangle = i\gamma_e P_p |A_p\rangle \quad \text{..................................(2)}
\]

\[
\frac{\partial |A_s\rangle}{\partial z} + \frac{i}{2} \Omega \mathbf{b} \bullet \mathbf{\sigma} |A_s\rangle = i\gamma_e P_p \left( 3 + \hat{P} \bullet \mathbf{\sigma} \right) |A_s\rangle \quad \text{..................................(3)}
\]

Where \( \Omega = \omega_p - \omega_s \) is the pump-probe frequency difference known as channel spacing, \( |A_p\rangle \) and \( |A_s\rangle \) are the Jones vectors of the pump and the probe respectively, \( \langle A_p | A_p \rangle \) is the pump power and \( \mathbf{\sigma} \) is the Pauli spin vector. Intra-channel PMD effects are included in \( \delta \). In (3), the Stokes vector of the pump is given by

\[
\hat{P} = \langle A_p | \mathbf{\sigma} | A_p \rangle P_p \quad \text{..................................(4)}
\]

The effective nonlinear parameter is \( \gamma_e \) and includes averaging over rapid variations of the pump SOP [7], [8]. In (3), the randomly varying vector \( \mathbf{b} \) accounts for residual birefringence and the same equation shows that the probe SOP changes randomly, at a rate determined by \( \Omega \mathbf{b} \). In the absence of the pump, linear birefringence changes the probe’s SOP randomly. When PMD is absent, the Kerr nonlinearities cause the probe’s phase to shift and induce nonlinear polarization rotation (NPR) of the probe. A combination of PMD and the Kerr effect causes the probe to become depolarized [7].

III. METHODOLOGY

Fig. 2: Set up: A two-channel system (PC = polarization controller)

The results discussed here were carried out using the setup shown in Fig. 2. A linearly polarized probe of input wavelength 1552.52 nm and a similarly polarized pump at input wavelength 1552.92 nm, giving a channel spacing of 50 GHz (0.4 nm) both in the form of continuous waves, were co-propagated in a 24 Km single mode fibre (SMF).

The standard single mode fibre had linear PMD of 0.5 ps/√km, effective area of 80 \( \mu \)m\(^2\) and a dispersion of 17 ps/nm-km. Polarization controller, PC1 maximized the probe input power into the fibre, while polarization controller PC2 rotated the input pump SOPs with respect to the probe signal SOP. The pump signal was launched at powers of 3 dBm, 7 dBm, 13 dBm and 16 dBm and for each input power the pump signal SOPs were rotated through 180° while observing the output DOP of the probe signal (Fig. 7). For the next simulation, the fibre was then modified into a concatenation of linearly birefringent trunks of constant length \( L_c \) to obtain different segments. Fig. 8 shows DOP of the probe against relative angles at constant pump power 13 dBm for 40 and 54 segments of the fibre.

For Matlab simulations, fibre of length 41 Km was modeled as a concatenation of linearly birefringent trunks with length \( L_c = 100 \) m and random birefringence axis orientation. At pump power of 1.9 mW and probe power of 0.005 \( \mu \)W the output signal is measured over time at different values of modal birefringence \( \Delta \beta_1 \) (Fig. 3) and the corresponding Poincare representation given in Fig. 4. Further simulations were done at a constant modal birefringence of \( \Delta \beta_1 = 0.3 \) ps/m and at a higher pump power of 0.18 W and low probe power of 0.005 \( \mu \)W the probe and pump output signals were measured over time (Fig. 5) and represented on the Poincare sphere as shown in Fig. 6.
IV. RESULTS AND DISCUSSION

Fig. 3: Effect of modal birefringence $\Delta \beta_1$ on the signal.

As evident in Fig. 3, the output signal varies in an oscillatory fashion and the signal broadens as the linear birefringence $\Delta \beta_1$ increases. The dependence of polarization fluctuations on the linear or intensity independent birefringence is supported by (1) and the oscillatory pattern seen in Fig. 3 shows that the signal simply rotates. The colours of the graphs in Fig. 3 correspond to those on the sphere and since linear polarizations take place at the equator then the overall result of birefringence is just a rotation of the signal State of Polarization (SOP) on the Poincare sphere as shown in Fig. 4.

Fig. 4: Poincare representation of Fig. 3.

Fig. 5: Effect of pump power on the probe signal.

As the pump power was increased to 0.18 W, there is more formation of peaks on the output probe signal (Fig. 5). This is because the induced nonlinear polarization rotations (NPR) due to high pump power distort the probe signal. Also, looking at the Poincare sphere (Fig. 6), there is no periodicity and the output signal scatters in most part of the sphere (DOP is randomized) implying that the signal is degraded.

Fig. 6: Poincare sphere representation

In Fig. 7, the rotation of the pump signal SOPs have a high effect on the probe signal as the relative angles between SOPs of probe and pump signal approach $90^\circ$, but the probe signal is less affected when the relative SOPs are launched parallel ($0^\circ$) or anti parallel ($180^\circ$) in Stokes space. When the input SOPs of the probe and pump signals were launched at $90^\circ$ from each other, the depolarization of the probe signal increased with increasing pump input power; however, if the launching angle was $0^\circ$ or $180^\circ$, increasing the pump power did not affect the probe signal. This implied that the non-linear birefringence penalty was high (low DOP) when the signal input SOPs in the two channels were perpendicular to one another in Stokes space (i.e. $45^\circ$ in Jones space). On the other hand, when the signal input SOPs were parallel or anti-parallel to each other in Stokes space (perpendicular in Jones space) the probe signal was less affected.

Fig. 7: Variation of probe DOP with relative angles for fibre length 24 Km.
space), the nonlinear birefringence penalty was very low. This is because at 90° the power is coupled equally into the two birefringent axes, and therefore the interaction is strong but when the relative signal SOPs are parallel or anti-parallel to each other, there is weak interaction between the signals resulting in a minimal nonlinear birefringence effect.

![Graph](image)

**Fig. 8**: Probe channel DOP variation as a function of relative SOP angle between the signals in the presence of linear and nonlinear PMD. The power for the probe signal was 3 dBm and that for the pump signal was 13 dBm.

From Fig. 8, it can be observed that the probe DOP variation was sinusoidal over the entire range of relative SOP angles. For instance, in the case of the 40 segment and 54 segment links, the probe SOPs had gone through nearly one cycle, while in the spool (black) alone the SOPs had covered half a cycle. At 90°, the DOP for the signal passing through the spool alone was low, while the DOPs of the signals passing through the 40 segments or 54 segments were relatively high. This was clear evidence that the linear PMD rotated the SOPs in a different direction to nonlinear PMD. In general, we deduce that in systems where both linear PMD and non-linear PMD exist, the DOP of the signal improves because the linear PMD cancels the nonlinear PMD up to second order. When the probe signal and pump signal SOPs were launched anti-parallel or parallel to one another, the nonlinear PMD had no effect on the probe signal.

V. CONCLUSION

PMD constitutes the major limiting factor of modern systems since it leads to broadening of the signal and rotation of signals SOPs. The effect of nonlinear birefringence-induced polarization rotation in a fibre depends on the orientation of the relative input polarization vectors of co-propagated channels and the power carried by each polarization vector. In low PMD links the nonlinear induced-birefringence effects scatter the SOPs thereby depolarizing the signal. In contrast, for links with high polarization mode coupling, the nonlinear birefringence sometimes couples with second-order PMD such that it reduces the penalty and improves the signal DOP. However, the interaction between the linear and nonlinear effects in the transmission system is still under study in order to utilize their potential to the fullest such as designing of fibre optic sensors and switches.

REFERENCES


