A Semi-Empirical Method for Determining the Averaged Orientation of Reinforcing Fibre in Fibre Reinforced Composites

Maina Maringa

Abstract—Fibre reinforced composites occur mainly in the form of longitudinally aligned continuous and discontinuous fibre, discontinuous randomly oriented and spatially distributed, fibre matts and fibre laminates. In these cases whilst design is reference to the categories given, re-orientation of the reinforcing fibres is likely to arise during placement of the fibres and compounding of the composite. The need does therefore arise for quick methods that can be used to verify conformity of the averaged fibre orientation with the design specifications. Expressions are developed here for use in predicting the averaged orientation of reinforcing fibre in fibre reinforced composites, based on known values of elastic modulus in any two mutually orthogonal directions. Theoretical validation of the expressions is carried out based on known properties in specific directions. Experimental validation however, is part of on-going work.

Keywords—Semi Empirical, Averaged Reinforcing Fibre Orientation, Elastic Modulus.

I. Introduction

Fibre reinforced composites occur in various forms depending on the type of constituents, as well as dimensions, orientation and spatial distribution of the reinforcing fibre. The reinforcing fibre occurs in fibre reinforced composites as aligned continuous and discontinuous, discontinuous randomly oriented and spatially distributed, fibre matts, and laminates as aligned continuous and discontinuous fibres [1, 2, 3, 4]. The reinforcing effect of fibres in a composite is a maximum and minimum in the longitudinal and transverse directions of the reinforcing fibres, respectively. This implies then that the manner in which the reinforcing fibres occur in fibre reinforced composites defines the mechanical properties of the composites [5, 6, 7]. Despite the maximum improvements of the properties of a composite being in the longitudinal direction of the reinforcing fibres, short fibre reinforced composites still find use due to advantages of lower production costs, ease of fabrication particularly of components with complex geometries, and macroscopic isotropy when the fibres are randomly oriented [5, 6, 7]. The values of the mechanical properties of fibre reinforced composites can be determined experimentally. They can also be predicted numerically or analytically. Analytical prediction of the mechanical properties of composites is done with differing degrees of accuracy based on the type of model used including; the Voigt rule (Rule of Mixtures – ROM), the Reuss rule (Inverse Rule of Mixtures – IROM), the Halpin Tsai Semi-empirical rules, micromechanics rules, and numerical models [1, 2, 3, 4]. The accuracy of predictions is dependent on the inputs into the rules or models used, including; the material properties and volume fractions of the constituent components, as well as the occurrence, length, orientation and spatial distribution of the reinforcing fibres [1, 2, 3, 5]. Application of these models to short fibre reinforced composites suffers the draw back that the models are based on the assumption that the reinforcing fibres are aligned parallel to one another [5] and parallel to or orthogonal to the direction of action of the applied load, a situation that is difficult to realise in practice [7]. Typically the assumption is made that the matrix is isotropic, and that the fibres are either isotropic or transversely isotropic, are the same in shape and size, and that the bond between the fibres and matrix is perfect [5, 6, 7].

The values of orientation [5, 7], length and spatial distribution of reinforcing fibres used in design do not apply wholly in practice, due to fibre breakage and re-orientation during formulation and moulding [1, 2, 6]. It is necessary therefore, to develop quick and accurate methods that can be used to verify conformity of the expected mechanical properties of the composite, with those specified in design. Such methods include ultrasonic, X-ray (tomography), SEM, Acoustic Emission, uniaxial tension, and transverse as well as longitudinal vibration testing [4]. Whilst ultrasonic, tomography and SEM are non-destructive, the cost of setting up units that can be used to scan whole components in industry is high and therefore, necessitates their alternative use in the laboratory on small test specimens sampled from large manufactured components. When applied this way, the methods are also destructive in the same way as tensile and vibration testing.

In this paper, the uniaxial tensile test is chosen to obtain data for use in determining the averaged orientation of fibre reinforced composites. The test is selected for its ease of application and familiarity, as well as the wide existence of tensile testing machines in material testing laboratories.

Nielsen and Chen (1968), proposed the following model for predicting the averaged elastic modulus ($\bar{E}$) of fibre reinforced composites, based on the directional values of elastic moduli (E_θ) for the composite [2].

$$\tilde{E} = (\int E_x d\theta)/(\int d\theta) \quad (1.0)$$

Maina Maringa, Department of Mechanical and Mechatronic Engineering, Central University of Technology, (+27 723432883; e-mail: maina_maringa@yahoo.com).

Tsai and Pagano (1968), proposed the following model for predicting the averaged elastic modulus ($\bar{E}$)) of a fibre reinforced composite, based on the values of the major and minor principle elastic moduli ($E_1$and E_2) of the composite[8].

$$\tilde{E} = \left(\frac{3}{5}\right) \mathrm{E}_1 + \left(\frac{2}{5}\right) \mathrm{E}_2 \quad (2.0$$

Christensen and Waals (1972) proposed the following equation for predicting the averaged elastic modulus of fibre reinforced composites [2].

$$\tilde{E} = \frac{1}{u_1}(u_1^2 - u_2^2)$$
(3.0)

In which the parameters ($u_1$and u_2) were defined as:

$$u_1 = \frac{3}{8}E_1 + \frac{G_{12}}{2} + \frac{(3+2\nu_{12}+3\nu_{12}^2)G_{12}K_{23}}{2(G_{23}+K_{23})}$$
(4.0)

$$u_2 = \frac{1}{8}E_1 - \frac{G_{12}}{2} + \frac{(1+6\nu_{12}+\nu_{12}^2)G_{12}K_{23}}{2(G_{23}+K_{23})}$$
(5.0)

Sharma (2001) and separately Vannan and Vizhian (2014) developed a semi-empirical model for use in predicting the generalised effective angle of orientation of reinforcing fibre in short fibre reinforced composites. The method involved first calculation of a theoretical value of elastic modulus for the composite (E_{ct}) based on an assumed orientation of the reinforcing fibre. Values of major and minor principle elastic moduli of the composite determined using the Voigt and Reuss rules were then calculated. The determined value of (E_{ct}) was then compared with an experimentally measured value of (E_{ce}) and the assumed angle of orientation of the reinforcing fibre adjusted iteratively, till the theoretical value (E_{ct}) and experimental value (E_{ce}) were equal to one another. The authors gave the following equation for use in determining the theoretical value of elastic modulus for the composite (E_{ct}).

$$E_c = \sqrt{(E_1 co\theta)} + 2(E_2 sin\theta)$$
(6.0)

The use of the Reuss rule by these authors in estimating the minor principle elastic modulus for the composite minimised the value of their method as the Reuss rule is known to give values that are significantly different from experimental ones. It was not possible to interrogate Equation 6.0 as is no details of its derivation were provided.

II. Theoretical Modelling

Using stress and strain tensors in laminate theory, Gibson (1994), presented the following equation predicting the *x*-direction value of elastic modulus (E_x) for fibre reinforced composites.

$$\frac{1}{E_x} = \frac{1}{\frac{cos^4\theta}{E_1} + \left(-\frac{\nu_{12}}{E_1} + \frac{1}{G_{12}}\right) + \frac{sin^4\theta}{E_2}}$$
(7.0)

In this model the *x*-direction elastic modulus is the elastic modulus of the composite in a direction that is inclined at an angle (θ) to the larger principle material direction of the fibre reinforced composite. The positive direction of the angel theta is measure counter clockwise from the *x*-axis direction.

Logically the *y*-direction elastic modulus will therefore be given by the expression:

$$\frac{1}{E_y} = \frac{1}{\frac{cos^4\left(\frac{\pi}{2}+\theta\right)}{E_1} + \left(-\frac{\nu_{12}}{E_1} + \frac{1}{G_{12}}\right) + \frac{sin^4\left(\frac{\pi}{2}+\theta\right)}{E_1}}$$
(8.0)

The cosine and sine terms are expanding to give:

$$cos\left(\frac{\pi}{2} + \theta\right) = cos\frac{\pi}{2}cos\theta - sin\frac{\pi}{2}sin\theta = -sin\theta \quad (9.0)$$

and

$$sin\left(\frac{\pi}{2} + \theta\right) = sin\frac{\pi}{2}cos\theta + cos\frac{\pi}{2}sin\theta = cos\theta \quad (10.0)$$

Therefore Equation 8.0 becomes:

$$\frac{1}{E_y} = \frac{1}{\frac{sin^4\theta}{E_1} + \left(-\frac{\nu_{12}}{E_1} + \frac{1}{G_{12}}\right) + \frac{cos^4\theta}{E_2}} \quad (11.0)$$

Subtracting Equation 11.0 from Equation 7.0 gives rise to:

$$\frac{1}{E_x} - \frac{1}{E_y} = cos^4\theta\left(\frac{1}{E_1} - \frac{1}{E_2}\right) + sin^4\theta\left(\frac{1}{E_2} - \frac{1}{E_1}\right) \quad (12.0)$$

Equation 12.0 can be written as:

$$\frac{1}{E_x} - \frac{1}{E_y} = (cos^4\theta - sin^4\theta)\left(\frac{1}{E_1} - \frac{1}{E_2}\right) \quad (13.0)$$

Factorising the cosine/sine term in Equation 13.0 gives rise to:

$$\frac{E_y - E_x}{E_x E_y} = (cos^2\theta - sin^2\theta)(cos^2\theta + sin^2\theta)\frac{E_2 - E_1}{E_1 E_2} \quad (14.0)$$

Introducing the cosine and sine identity, $cos^2\theta + sin^2\theta = 1$, into Equation 14.0 leads to:

$$\frac{E_y - E_x}{E_x E_y} = \left(cos^2\theta - (1 - cos^2\theta)\right) \times \left(cos^2\theta + (1 - cos^2\theta)\right)\frac{E_2 - E_1}{E_1 E_2} \quad (15.0)$$

Equation 15.0 reduces to:

$$\frac{E_y-E_x}{E_xE_y} = (2cos^2\theta - 1)(1)\frac{E_2-E_1}{E_1E_2} \quad (16.0)$$

Making the cosine term the subject in Equation 16.0 leads to:

$$2cos^2\theta = 1 + \frac{E_y-E_x}{E_xE_y} \times \frac{E_1E_2}{E_2-E_1} \quad (17.0)$$

Taking square roots on both sides of Equation 17.0 and placing the cosine term in the arising equation on one side gives rise to:

$$cos\theta = \sqrt{0.5\left(1 + \frac{E_y-E_x}{E_xE_y} \times \frac{E_1E_2}{E_2-E_1}\right)} \quad (18.0)$$

The averaged orientation of the reinforcing fibre is therefore given by:

$$\theta = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_y-E_x}{E_xE_y} \times \frac{E_1E_2}{E_2-E_1}\right)}\right] \quad (19.0)$$

The averaged orientation of reinforcing fibre in a fibre reinforced composite can now be determined from Equation 19.0, given known values of the principle elastic moduli and the *x*- and *y*-direction elastic moduli.

The Rule of Mixtures (RoM) or Voigt rule is still the best method for calculating values of major principle elastic moduli for fibre reinforced composites. It represents the upper bound for composites and is recommended for use here. A number of methods exist for predicting values of minor principle elastic moduli for fibre reinforced composites. These include, the Inverse Rule of Mixtures (IRoM) or Reuss rule, the Chamis simplified micromechanics rules, and Gibson's representative volume element, strain energy equations. The IRoM is the most inaccurate of these four methods and is not therefore recommended for us here. It represents the lower bound for composite materials [1, 2, 3]. The Halpin-Tsai semi-empirical method is the most widely used of these methods [2] and is recommended for use in determining values of minor principle elastic moduli.

It is proposed that the x- and y-direction elastic moduli are determined experimentally using the uniaxial tensile tests. Test specimens for use in determining the values of ($E_x$and E_y) should be obtained from a parent fibre reinforced composite sheet, with their longitudinal axes oriented orthogonal to one another. It is recommended that a large number (at least 10) of mutually orthogonal test sample pairs with the same orientation be used for each orientation. The orientation should then be varied a number of times (at least 10), and with each variation 10 mutually orthogonal test specimen pairs are obtained. The samples for testing should be obtained from the entire parent sheet in order to test for spatial variability of the averaged orientation of the reinforcing fibre. This will ensure availability of adequate data at any one orientation and in an adequate number of different orientations, from which inferences on trends can be made.

Because of the orthotropic nature of fibre reinforced composites it is necessary to be aware of the shear coupling effect whose magnitude increases with decreasing length to width ratio of a specimen. Short specimens that are thick should be avoided and rather slender specimens with long distances between the test specimen clamps are preferred. The testing frame grips should also be made flexible in order to avoid introducing distortion in the specimen due to the arising end constraints [7].

III. THEORETICAL VALIDATION OF THE PROPOSED MODEL

Consider the case where *x*-direction coincides with the direction of the major principle material axis. In such a case the *x*-direction elastic modulus (E_x) will be equal to the major principle elastic modulus (E_1). Moreover, the y-direction elastic modulus (E_y) will be equal to the minor principle elastic modulus (E_2). Substituting this condition into Equation 19.0 gives rise to:

$$\theta = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_2-E_1}{E_1E_2} \times \frac{E_1E_2}{E_2-E_1}\right)}\right] = cos^{-1}\left[\sqrt{0.5(1+1)}\right] \quad (20.0)$$

$$\theta = cos^{-1}[1] = 0^0 \quad (21.0)$$

Equation 21.0 conforms to the starting assumption that the x-direction elastic modulus is inclined at an angle ($\theta = 0^0$) to the direction of the major principle material axis.

Now consider the case where *x*-direction coincides with the direction of the minor principle material axis. In such a case the *x*-direction elastic modulus (E_x) will be equal to the minor principle elastic modulus (E_1), while the *y*-direction elastic modulus (E_y) will be equal to the major principle elastic modulus (E_2). Substituting this condition into Equation 19.0 gives rise to:

$$\theta = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_1-E_2}{E_2E_1} \times \frac{E_1E_2}{E_2-E_1}\right)}\right] = cos^{-1}\left[\sqrt{0.5(1-1)}\right] \quad (22.0)$$

$$\theta = cos^{-1}[0] = 90^0 \quad (23.0)$$

Equation 23.0 conforms to the starting assumption that the *x*-direction elastic modulus is inclined at an angle ($\theta = 0^0$) to the direction of the minor principle material axis, and thus an angle ($\theta = 90^0$) to the major principle material axis.

Finally consider the case where *x*-direction is inclined at an angle of 45^0 to the direction of the major principle material axis. In such a case the *x*-direction elastic modulus (E_x) will be equal to the *y*-direction elastic modulus (E_y). Substituting this condition into Equation 19.0 gives rise to:

$$\theta = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_x-E_x}{E_xE_x} \times \frac{E_1E_2}{E_2-E_1}\right)}\right] = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_y-E_y}{E_yE_y} \times \frac{E_1E_2}{E_2-E_1}\right)}\right] \quad (24.0)$$

$$\theta = cos^{-1}\left[\sqrt{0.5(1+0)}\right] = cos^{-1}\left[\sqrt{0.5(1+0)}\right] \quad (25.0)$$

$$\theta = cos^{-1}\left[\sqrt{0.5}\right] = 45^0 \quad (26.0)$$

Equation 26.0 conforms to the starting assumption that the x-direction elastic modulus is inclined at an angle ($\theta = 45^0$) to the direction of the major principle material axis.

IV. Referencing the Present Model to the Material Properties of the Constituent Components

The major principle elastic modulus (E_1) is determined from the Rule of Mixtures as:

$$E_1 = E_f\upsilon_f + E_m\upsilon_m \quad (27.0)$$

The minor principle elastic modulus (E_2) is determined from the Halpin-Tsai semi-empirical equations, which have been re-organised here as:

$$E_2 = \left[\frac{E_f E_m(E_f + E_m\xi) + (E_f - E_m)\xi\upsilon_f E_m^2}{E_f(E_f + E_m\xi) - (E_f - E_m)E_m}\right] \quad (28.0)$$

Adopting a value of ($\xi = 2$) [3] and re-organising the arising equation gives rise to:

$$E_2 = -E_m\left[\frac{2E_m^2\upsilon_f - 2E_mE_f(1+\upsilon_f) - E_f^2}{E_m^2 + E_fE_m + E_f^2}\right] \quad (29.0)$$

Substituting Equations 28.0 and 29.0 into Equation 19.0 gives rise to:

$$\theta = cos^{-1}\left[\sqrt{0.5\left(1 + \frac{E_y - E_x}{E_xE_y} \times \frac{E_m\left[E_f\upsilon_f + E_m\upsilon_m\right]\left[\frac{2E_m^2\upsilon_f - 2E_mE_f(1+\upsilon_f) - E_f^2}{E_m^2 + E_fE_m + E_f^2}\right]}{E_m\left[\frac{2E_m^2\upsilon_f - 2E_mE_f(1+\upsilon_f) - E_f^2}{E_m^2 + E_fE_m + E_f^2}\right] + (E_f\upsilon_f + E_m\upsilon_m)}\right)}\right] \quad (30.0)$$

Equation 30.0 is now in a form that facilitates direct substitution of the commonly available values of material properties for the constituent components of a composite namely, the fibre and matrix elastic moduli and volume fraction.

Equations 19.0 or 30.0 are variants of one another and both give rise to the same result.

V. Flow Chart for the Process

The semi-empirical method of determining the averaged orientation of reinforcing fibre in a fibre reinforced composite using Equations 19.0 and 30.0, separately, is presented in the form of a chart in Figure 1. It is evident from the equations and the flow chart that the semi-empirical method presented here is amendable to use with modern day software. Such software will have a program developed to receive basic data on the material properties of the constituents of a fibre reinforced composite and to then calculate values of principle elastic moduli. The program also receives experimental data on the x- and y-direction values of composite elastic moduli, which is combined with the principle values of elastic moduli to produce values of averaged reinforcing fibre orientation.

The flow chart shown in Fig. 1 describes the semi-empirical method proposed here for use in the determination of the averaged fibre orientation of a fibre reinforced composite.

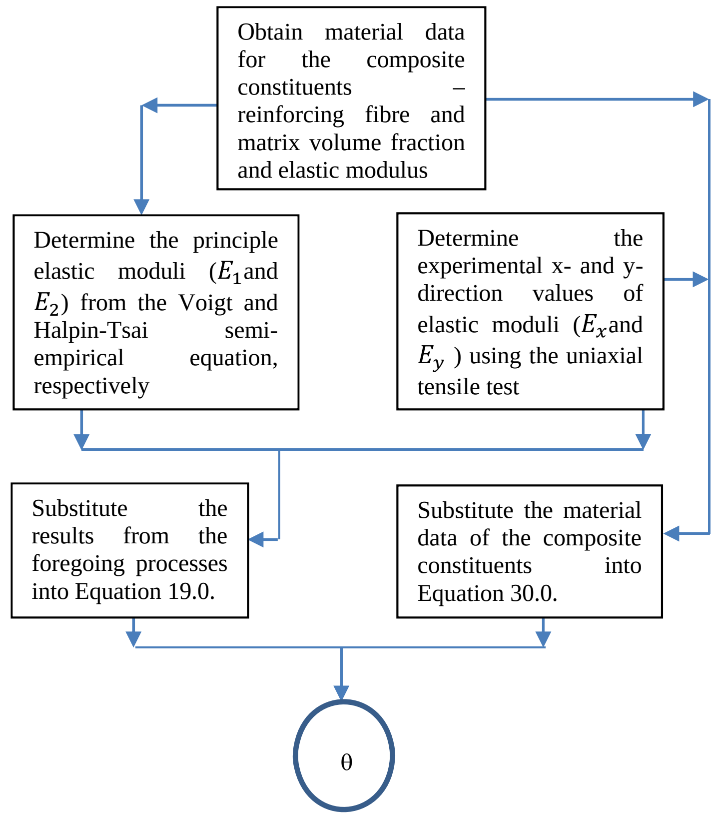

Fig. 1: Flow Chart of the Semi-Empirical Methods Presented Here

VI. Conclusion

A semi-empirical model has been developed for use in determined the averaged orientation of reinforcing fibre in a fibre reinforced composite.

Theoretical validation of the model has been undertaken successfully based on known results in three different orientations.

VII. Recommendations

It is recommended that experimental testing be carried as outlined in this paper out in order to provide validation for the model against actual composites.

References

[1] Derek Hull, “An Introduction to Composite Materials”, Cambridge University Press, London. 1990.

[2] Gibson R. F., “Principles of Composite Material Mechanics”, McGraw Hill Inc., Copyright ©, 1994.

[3] Harris Bryan, “Engineering Composite Materials”, Institute of Materials, London, 1999.

[4] Department Of Defence Handbook, Composite Materials Handbook"- Volume 5. Ceramic Matrix Composites", MIL-HDBK-17-5, Volume 5 of 5, 17 JUNE 2002.

[5] Sharma S. C., "Elastic Properties of Short Glass Fiber-Reinforced ZA-27 Alloy Metal Matrix Composites", Journal of Materials Engineering and Performance, Volume 10(4), August 2001, pp 468 – 474.

[6] Pan N., "The Elastic Constants of Randomly Oriented Fibre Composites: A New Approach to Prediction", Science and Engineering of Composites, Vol. 5 No. 2, 1996, pp 63 – 72.

[7] Vannan E and Vizhian P., "Prediction of the Elastic Properties of Short Basalt Fibre Reinforced Al Alloy Metal Matrix Composite", Journal of Minerals and Materials Characterisation and Engineering, 2014, 2, 61-69

[8] Pagano N. J. and Halpin J. C., "Influence of end Constraints in Testing Orthotropic Bodies", Journal of Composite Materials, Vol. 2, No. 1 (January 1968), pp 18 – 31.