

# Optimal Placement of FACTS Devices using Voltage Stability Indices

Mutegi A. M, Kihato P. K, Muriithi C. M and Saulo M. J

**Abstract**— Power System Voltage stability remains a major challenge for power utilities across the world. This research uses static load flow methods namely the voltage stability indices to come up with an optimal placement of FACTS devices in the weak and heavily loaded buses for voltage stability improvement. The research shall be done by the use of Power System Analysis Toolbox (PSAT) software that runs on MATLAB's environment on the IEEE 39-Bus 10-Generator test system.

**Keywords** - FACTS, Optimal, Security-Constrained and Versatile

## I. INTRODUCTION

The demand for clean, reliable and affordable energy is growing at unprecedented rates across the world. As we seek to increase the quantity of energy produced, the supply quality challenges will grow in tandem. The growing long distances between generation and load centers only serve to compound the voltage stability challenge.

This presents power system engineering practitioners with the challenge of coming up with ways and means of maintaining the required system voltage profiles. Voltage stability is the ability of a power system to maintain acceptable voltage levels under normal operating conditions and after being subjected to disturbances such as a sudden increase in load[1].

Research on the location of the FACTS devices using such methods as small signal analysis, hopf bifurcation, time domain analysis, loss sensitivity factors, fuzzy index and voltage change index have been well documented. This has been coupled with various FACTS devices control strategies such as genetic algorithm, particle swarm optimization, pulse width modulation and runge-Kutta method [2][3][4][5].

Various methods are used for voltage stability studies. They can broadly be classified into two, namely the static and dynamic methods. Dynamic methods apply real-time simulation in time domain using precise dynamic models. Static methods solve specific first or second order functions or indices derived from the power flow equations of the network

which show the capability of the power system to remain stable. They run with specific load increases until the voltage collapse point is reached thus allowing the examination of a wide range of system operating conditions such as heavy loading and contingencies. The objective of this study is to use the voltage stability indices in predicting the proximity to voltage collapse as one of the static methods[6][7][8].

Proper knowledge of how close the actual system's operating voltages are from the voltage stability limits is crucial to system operators, power system engineers and other practitioners. Therefore voltage stability indices are crucial parameters for many voltage stability studies. These indices provide critical information about the proximity of voltage instability in a power system. Their values change between 0 (no load) and 1 (voltage collapse)[9].

Two versatile voltage stability indices, namely the Line stability Index and the Fast Voltage Stability Index shall be used in this research to locate the optimal-practical and cost effective- placement of FACTS devices in the weak and heavily loaded buses. The research shall also take care of the objective of system voltage security thus the use of voltage security-constrained load flow analysis as the base case.

The paper is organized as follows: Section 1 serves as an Introduction. Section 2 discusses the Mathematical Modelling of the Voltage Stability indices used and in section 3, we discuss the methodology used and finally section 4 looks at the results obtained.

## II. VOLTAGE STABILITY INDICES

One area of static methods is called the Voltage Stability Index (VSI) method. The voltage stability indices are generated from the power flow equations. The indices show the system's stability condition and can be used to estimate the systems operating states[10].

The mathematical expression of a VSI is written as a polynomial containing the systems real-time measurements such as voltage magnitudes, phase angles, bus injected power and branch power flow values.

The values of VSI are distinctly different in normal condition and contingencies for a power system. The changing of the VSI values from no load condition to maximum permissible loading condition reflects the system's stability trend from

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stable to unstable. The point when the system loses stability is called the optimal (diverge) point.

Voltage magnitude is the most often used parameter in voltage stability index studies. A typical Voltage Stability Analysis considering voltage magnitude is based on a simplified 2-bus thevenin Equivalent power system with line resistance neglected.

The approximate power flow equations through sending and receiving ends are obtained.

One VSI method considering voltage magnitude of the receiving end is derived. The method utilizes the approximation of neglecting the line resistance for transmission lines with a high reluctance/resistance ratio. The approximated maximum active/reactive and apparent power flow values are obtained by using power flow measurements to express the voltage magnitude at receiving end and calculating its minimum value.

The Fast Voltage Stability Index – FVSI-is an indicator based on measurements of voltages and reactive power. Its a very good indicator of the weakest lines in the network for mitigation such as placement of FACTS devices[11][12][13][14][15][16].

The line model used to derive the indicator is shown below:

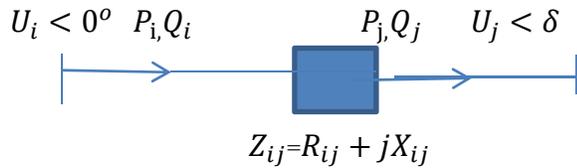


Fig.1 Two-Bus Line Model for derivation of Voltage Stability Indices

We have two Buses namely Bus i (sending) and Bus j (receiving).  $U_i$  is the sending end voltage,  $Q_j$  the reactive power at the receiving end. Bus i is used as the reference bus, with the voltage angle set to 0.

The derivation of the index begins with the general equation for the current in a line between two Buses i and j as:

$$I_{ij} = \frac{U_i - U_j}{Z_{ij}} \quad (1)$$

The apparent power received at Bus j is found by multiplying 1 with the voltage at Bus j as:

$$I_{ij} = U_j \frac{U_i - U_j}{Z_{ij}} = P_j + jQ_j \quad (2)$$

The imaginary part of 2 is the reactive power received at Bus j given by equation:

$$Q_j = \frac{U_i U_j (R_{ij} \sin \delta + X_{ij} \cos \delta) - X_{ij} U_j^2}{R_{ij}^2 + X_{ij}^2} \quad (3)$$

This can be rewritten as a second-order equation for  $U_j$  as:

$$U_j^2 - U_j U_i \left( \frac{R_{ij}}{X_{ij}} \sin \delta + \cos \delta \right) + \left( X_{ij} + \frac{R_{ij}^2}{X_{ij}} \right) Q_j = 0 \quad (4)$$

FVSI is based on the principle that the system remains stable as long as there are only real solutions to equation 4 above. i.e.

$$\left[ \left( \frac{R_{ij}}{X_{ij}} \sin \delta + \cos \delta \right) U_i \right]^2 - 4 \left( X_{ij} + \frac{R_{ij}^2}{X_{ij}} \right) Q_j \geq 0 \quad (5)$$

Simplifying 5 above and assuming that the angle difference  $\delta$  is normally very small ( $\delta \approx 0$ ,  $R_{ij} \sin \delta \approx 0$  and  $X_{ij} \cos \delta \approx X_{ij}$ ) gives:

$$(X_{ij} U_i)^2 \geq 4 X_{ij} + R_{ij}^2 Q_j \quad (6)$$

FVSI is thus defined as the ratio between the two terms:

$$FVSI = \frac{4 Z_{ij}^2 Q_j}{U_i^2 X_{ij}} \leq 1 \quad (7)$$

As shown in equation 7, the power transmission through line i-j is stable as long as

$$FVSI \leq 1$$

Line stability Index  $L_{mn}$  resembles FVSI based on the power flow equation for a transmission line. Continuing from equation 3 above and replacing  $R + jX$  by  $Z < \theta$ , gives an expression for the received reactive power at Bus j:

$$Q_j = \frac{U_i U_j}{Z_{ij}} \sin(\theta - \delta) - \frac{U_j^2}{Z_{ij}} \sin \theta \quad (8)$$

Using the same technique as for FVSI, the receiving-end voltage can be expressed as a second-order equation:

$$U_j^2 \sin \theta - U_j U_i \sin(\theta - \delta) - Q_j Z_{ij} = 0 \quad (9)$$

Similarly as with FVSI, the system remains stable as long as there are only real solutions to equation 9 above.

Rearranging the equation and using the fact that  $Z_{ij}\sin\theta = X_{ij}$  gives the equation for the line stability as:

$$L_{mn} = \frac{4X_{ij}Q_j}{U_i^2 \sin^2(\theta - \delta)} \leq 1 \quad (10)$$

The similarity of the two indicators can be illustrated by inserting  $\delta = 0$  in equation 10 above to give:

$$L_{mn} = \frac{4X_{ij}Q_j}{U_i^2 \sin^2(\theta)} = \frac{4Z_{ij}Q_j}{U_i^2 X_{ij}} = FVSI \text{ for } \delta = 0 \quad (11)$$

Thus the only difference between  $L_{mn}$  and FVSI is that  $L_{mn}$  accounts for the voltage angle difference which FVSI assumes to be zero. This is also the advantage of FVSI over  $L_{mn}$  as it only requires measurements of magnitudes only whereas  $L_{mn}$  requires synchronized phasor measurements at both line ends.

### III. METHODOLOGY

We shall use the voltage security constrained optimal load flow solution on the IEEE 10 Generators 39 bus system as shown in Fig.A1 below as our base case.

The IEEE 10 Generators 39 bus system is first drawn on the MATLAB's Simulink and run on the PSAT software to obtain the base case load flow solution as outlined above.

This will assist to achieve the twin goals of economy and security of the power system. In this research the constraints are the minimization of real power losses, minimization of the cost of active power generation, minimization of reactive power losses for better voltage profiles, maximization of active power transfers and minimizing the cost of installation of the FACTS devices. All voltage profiles have been limited to 0.9 to 1.1 per unit values for enhanced voltage security.

The load flow equations shall further be enhanced so as to obtain the voltage stability indices, namely the Fast Voltage Stability Index and the Line Stability Index.

The above voltage stability indices shall be obtained using the steps summarized below:

- First run the load flow program for the base case as above.
- Next, evaluate the FVSI value for every line in the system.
- Gradually increase the reactive power loading by 0.01 per unit at a chosen load bus until the load flow solution fails to give results for the maximum computable FVSI and extract the stability index that has the highest value.
- Choose the next load bus and repeat the above two steps for all the nineteen load buses.
- Finally, sort the maximum loadability- maximum reactive power loading- of all the load buses in

ascending order with the smallest maximum loadability ranking as the highest which implies the weakest bus in the system.

### IV. RESULTS

The load flow solution for the base case converged in 0.388 seconds as shown below:

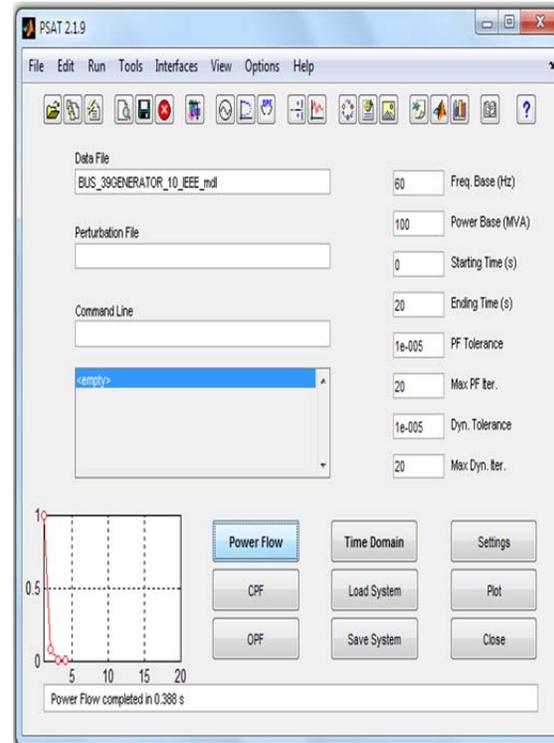


Fig.2 Load flow solution of the IEEE 39-Bus 10-Generator System

The ranking system below was developed to determine the stressed and most heavily loaded load buses:

Table I: Load Bus Ranking using the FVSI and  $L_{mn}$  Techniques

RANK	LOAD BUS	FVSI	$L_{mn}$
1	3	0.9224	0.9429
2	4	0.9125	0.9401
3	7	0.9115	0.9385
4	8	0.9075	0.9276
5	16	0.9029	0.9165
6	18	0.9009	0.9111
7	15	0.8997	0.9008
8	20	0.8901	0.8997
9	21	0.8899	0.8966
10	12	0.8865	0.8911
11	24	0.8725	0.8895
12	39	0.8711	0.8856
13	25	0.8615	0.8735
14	26	0.8556	0.8662
15	27	0.8433	0.8595
16	31	0.8256	0.8487
17	28	0.8166	0.8345
18	23	0.8127	0.8276
19	29	0.8066	0.8215

## V. DISCUSSION

Using the Voltage security constrained optimal load flow solution alongside (7) and (10) above we computed the FVSI and  $L_{mn}$  respectively for all the nineteen load buses. The results are ranked as per table I above.

From the above table, Load Bus#3 is the weakest bus thus the best candidate for optimal placement of FACTS devices for voltage stability improvement. On the other hand, Bus#29 is the most stable buses and the last candidate for voltage support using FACTS devices.

From (10) and (11) above, the results of  $L_{mn}$  are more sensitive than those of FVSI in identification of the weakest buses. This is because  $L_{mn}$  accounts for the voltage angle difference which FVSI assumes to be zero. The results in table I above confirms the said assertion.

Continuous monitoring should be done on the weak buses so that the load connected to them does not exceed maximum allowed limit for system stability. Further, voltage support by the use of FACTS devices will go a long way in improvement of system voltage stability.

The sparse matrix visualization below further confirms the above results:

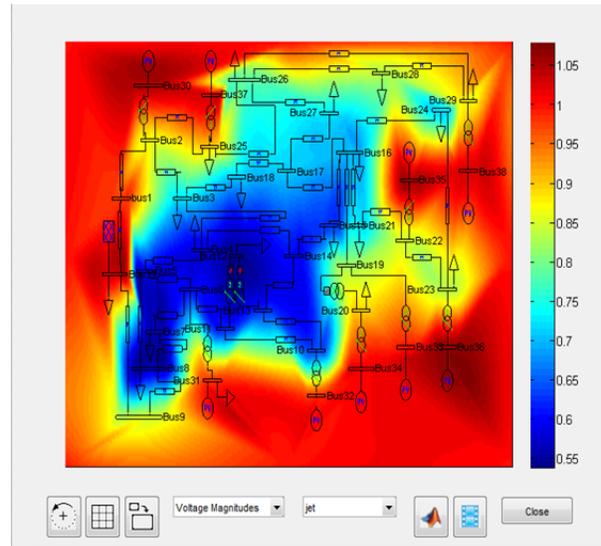


Fig.3 Sparse Matrix visualization of the IEEE 39-Bus 10-Generator System

## VI. CONCLUSION

A successful analysis of the IEEE 39-Bus 10-Generator system for identification of weak load buses was carried out.

Voltage security constrained optimal load flow solution was used as the base case on which the research was built on.

The two voltage stability indices, namely the Fast Voltage Stability Index and the line stability index helped to determine the maximum load that can be connected to a load bus so as to maintain system voltage stability.

## REFERENCES

- [1] P.Kundur, "Control of Active Power and Reactive Power," in *Power System Stability and Control*, 2<sup>nd</sup>ed. British Columbia, Canada: McGraw-Hill Inc., 1994, ch. 14, sec. 1, pp. 959-969.
- [2] Vibhor Gupta, "Study and Effects of UPFC and its control system for Power Flow control and Voltage Injection in a Power System," *Journal of Eng.Sc. and Tech.*, vol. 2, no. 7, pp. 2558-2566, Jul. 2010.
- [3] Panumat Sanpoung, "Analysis and control of UPFC for voltage compensation using ATP/EMTP," *Asian Journal on Energy and Environ.*, vol. 10, no. 4, pp. 241-249, Jun. 2009.
- [4] P.S.Venkataramu and T.Ananthapadmanabha "Installation of unified power flow controller for voltage stability margin enhancement under line outage contingencies," *Iranian Journal of Elect. and Computer Eng.*, vol. 5, no. 2 pp. 90-96, Dec. 2006.
- [5] K Manoj Kumar Reddy, "Transmission Loss Minimization Using Advanced Unified Power Flow Controller-UPFC," *IOSR Journal of Eng.*, vol. 2, no. 2, pp. 1049-1052, May 2012.

- [6] Mohammad Nizan, "Performance Evaluation of Voltage Stability Indices for Dynamic Voltage Collapse Prediction," *Journal of Appl. Sci. Res.* vol.1, no. 1, pp.1104-1113, Apr.2006
- [7] Christine E.Doig, "Analysis on Voltage Stability Indices," M.Sc.Thesis,Inst.Aut.for Compl.Power Sys.,Aachen Univ.,Aachen,Springer-Velag,2011.
- [8] *Power System Analysis*,2<sup>nd</sup> ed.,Hyderabad,2007, pp 205-299.
- [9] D.P.Kothari, "Voltage Stability," in *Modern Power System Analysis*,3<sup>rd</sup>ed.,Delhi:McGraw-Hill Inc.,2003,ch.17,pp.591-601.
- [10] Jan Machowski, "Voltage Stability," in *Power System Dynamics:Stability and Control*,2<sup>nd</sup>ed.,The Atrium,United Kindgom:John Willey & Sons Inc.,1994,ch.8, pp.299-334.
- [11] K.R Vadivelu and Dr G.V Marutheswar, "Fast Voltage Stability Index based Optimal Reactive Power Planning using Differential Evolution,"*Int. Journal of Elect. Eng. Res.*vol.3,no. 1, pp.236-245, Apr.2014.
- [12] Sahar Asala and Alireza Gorzin, "Comparison of Voltage Stability Indicators in Distribution Systems,"*Indian Journal of Scie.*vol.2,no. 1, pp.5-10,Jun.2014.
- [13] Mayur D., "Optimal and Fast Placement of DG unit CPF Method using MATLAB toolbox PSAT,"*Int. Journal of Eng. and Res.*vol.2,no.2,pp.341-349,Jun.2014.
- [14] K R Vadivelu and G V Marutheswar, "Maximum Loadability Estimation for weak bus Identification using Fast Voltage Stability Index in a power Transmission System by real-time Approach,"*Int. Journal of Elec.and Electr. Eng. and Tel.*vol.3,no.1,pp.388-398,Jan.2014.
- [15] Pinki Yadav and P R Sharma, "Enhancement of Voltage Profile for IEEE 14-Bus System by using Static Var Compensation,"*Journal of Elec.and Electr. Eng.*vol.9,no. 1, pp.76-81,Jan.2014.
- [16] Rajesh Ahuja and Shakti Vashisth, "Simulation of real and reactive power flow control with UPFC connected to a Transmission Line,"*Int.Journal of Innovative res. and studies*, vol.3,no.1,pp.235-253,Jun.2014.

APPENDIX

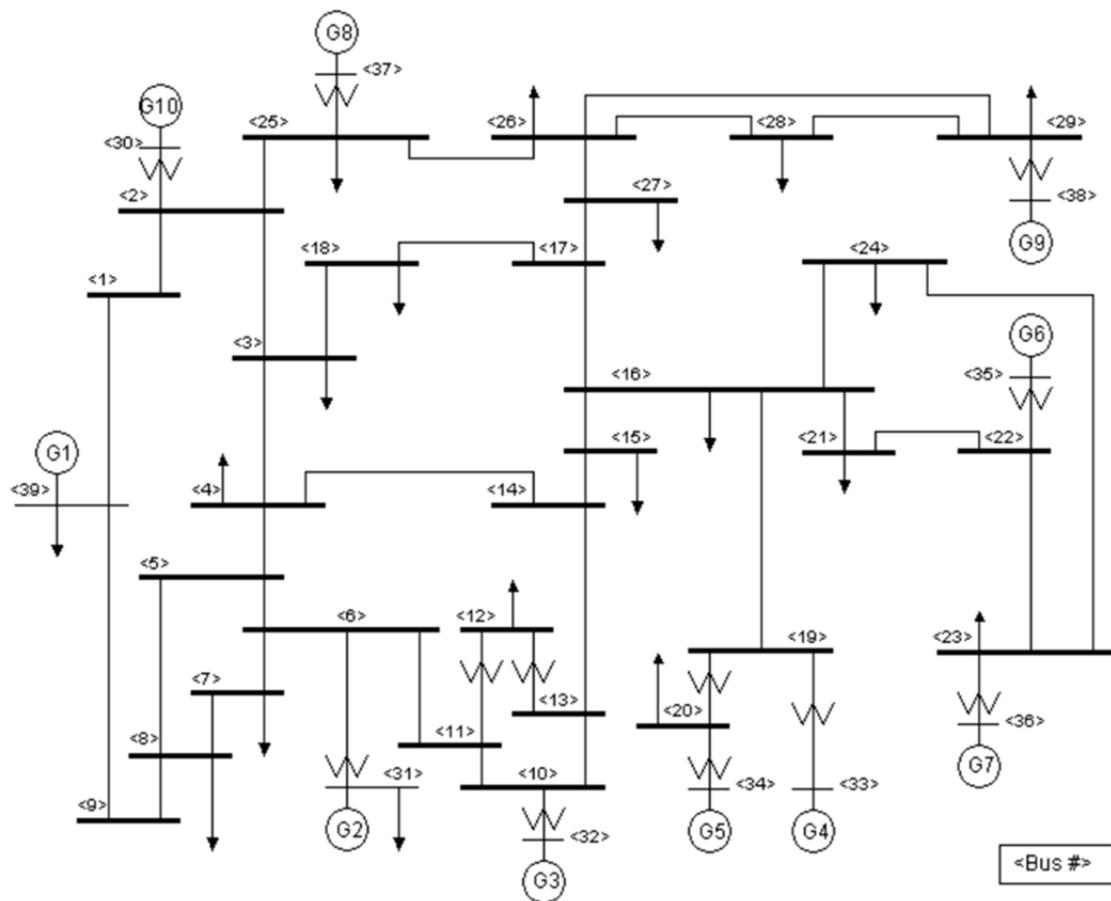


Fig.A1 The IEEE 10 Generators 39 bus system