Reference Signal based Beamforming vis-à-vis Null Steering based Beamforming on the basis of a Rectangular Antenna Array

Robert Macharia Maina, Kibet Langat and P. K. Kihato

***Abstract*—**Smart antennas optimize radiation/ reception patterns (beamforming) in accordance to particular approaches. The common approaches utilized include beam-steering, null-steering, reference signal based approaches and statistical blind approaches. Null and beam steering criterions are viable in situations in which desired maximal and minimal radiation directions are known. Reference signal criterions are viable in situations in which a signal known at both ends of a wireless link is present. Some situations involve the presence of all requisite information as far as null-steering and reference signal based beamforming approaches are concerned. This paper takes a look into such a scenario, comparing the viability of either of the two approaches. A rectangular antenna array is utilized in reception mode. Situations featuring a single desired reception direction and four interferer directions are considered. Reception beamforming is done on the basis of the null-steering and reference signal based approaches separately. Particle Swarm Optimization (PSO) algorithm is utilized in solving the two beamforming approaches. The resultant reception patterns and the correlation values between the beamformed signal and the expected signal are used as performance measures. The reference signal based beamforming approach is found better than the null steering based approach on the basis of the utilized performance measures.

***Keywords*—**Smart antenna, Rectangular antenna array, Beamforming, Reference signal, Beam/ null steering

I. INTRODUCTION

Common smart antennas beamforming criterions include: beam-steering, null-steering, reference signal based approaches and statistical blind approaches.

Beam steering is a typical approach to beamforming, involving orientation of the main radiation lobe towards a desired radiation direction. Null steering further demands that nulls are enforced in undesired radiation directions. Reference signal based beamforming makes use of a signal that is known at both ends of a wireless communication link [1], [2], [3], [4].

Common antenna arrangements utilized in smart antennas are linear, circular and rectangular. The rectangular antenna arrangement is usually the preferred option in situations requiring 3-dimension beamforming.

Mr. Robert Macharia, Department of Telecommunication Engineering, JKUAT. (phone: +254728935662; e-mail: robertisaacm@gmail.com).
Dr. Kibet Langat, Department of Telecommunication Engineering, JKUAT.
Dr. P. K. Kihato, Department of Electrical Engineering, JKUAT.

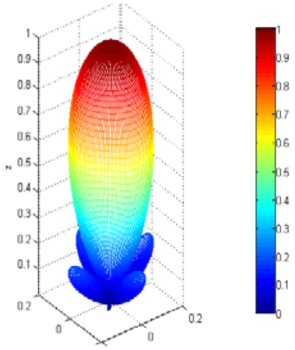

Fig. 1 An illustration of beamformed radiation

II. BEAM/ NULL STEERING

An antenna array beamformer automatically adapts its response to different scenarios. A criterion has to be defined to allow adaption to take place. In this paper, the criterion of choice is the beam/ null steering approach. Fig. 2 below is an illustration of a beam/ null steering based adaptive beamformer.

The weights of the null steering beamformer are chosen to synthesize a beam with unit gain in the direction of the desired signal and nulls in the direction(s) of interferer(s) [5].

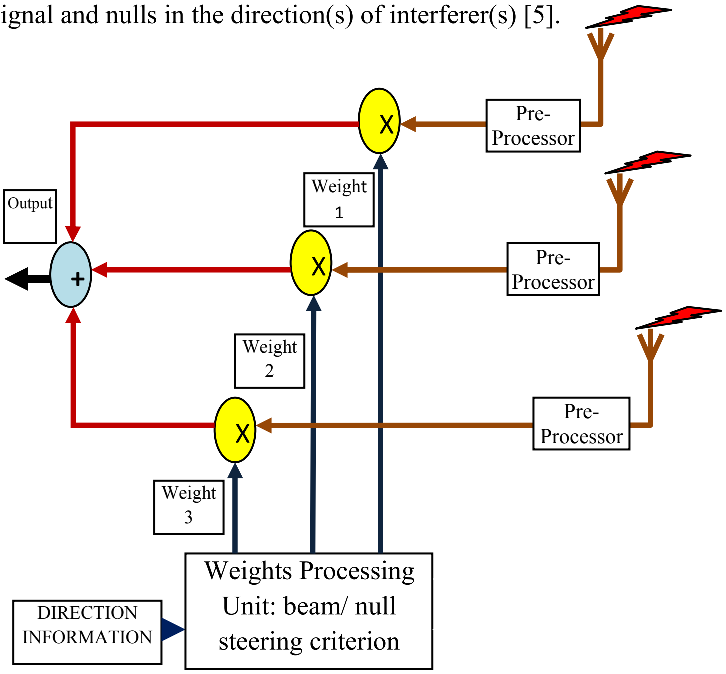

Fig. 2 Beam/ Null Steering based Adaptive Beamformer

III. REFERENCE SIGNAL BASED BEAMFORMING

Reference signal based beamforming translates to a Minimum Mean Square Error (MMSE) problem. The weights of the MMSE based beamformer are synthesized to produce minimum difference between a desired signal or a reference signal closely representing the desired signal and the output of the beamformer[4]. This yields an optimal output. The MMSE scheme requires the knowledge of the desired signal or a closely correlated replica to use as the reference signal. MMSE beamforming is generally computationally intensive. The mean square error is evaluated as depicted in (1), (2) and (3).

$$\varepsilon(t) = r(t) - y(t) \qquad (1)$$

$$\varepsilon(t) = r(t) - W^H X(t) \qquad (2)$$

$\varepsilon(t)$ denotes instantaneous error, the instantaneous reference signal quantity, the instantaneous received signal quantity, the Hermitian transpose of array weights vector and the instantaneous signal quantity at each array element.

The mean squared error is given by:

$$\epsilon(W) = E[|\varepsilon(t)|^2] = E[\varepsilon(t)\varepsilon^*(t)] \qquad (3)$$

Substituting (2) into (3);

$$\epsilon(W) = E[|r(t)|^2] + W^H R_{xx} W - W^H Z - Z^H W \qquad (4)$$

Where $Z = E[X(t)r^*(t)]$ is the correlation between the reference signal and array signal vector. The mean square error (MSE) surface is a quadratic function of W and can be minimized by setting its gradient with respect to W equal to zero, with its solution yielding the optimal weight vector.

Fig. 3 below is an illustration of a reference signal based adaptive beamformer.

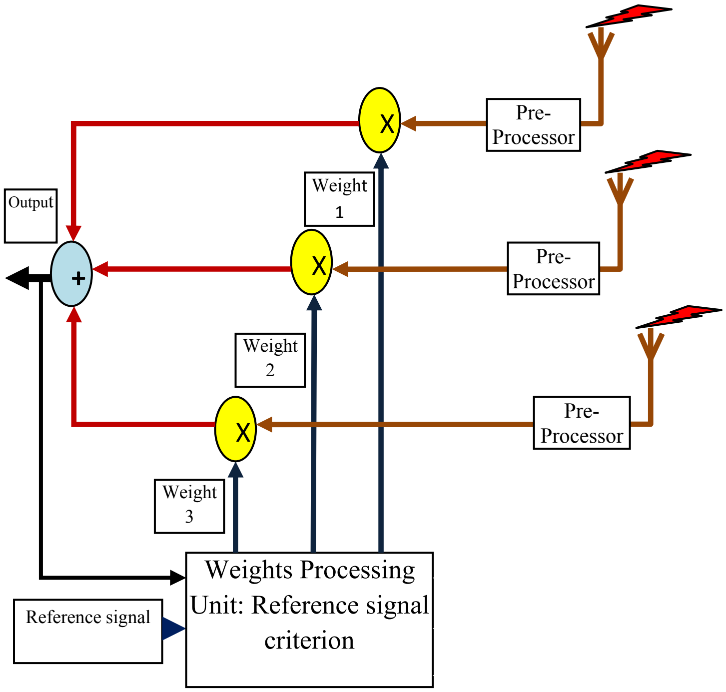

Fig. 3 Reference signal based Adaptive Beamformer

IV. RECTANGULAR ANTENNA ARRAY

Fig.4 below depicts an M by N rectangular array with uniform element spacing distance. A planar wave front impinges on the array.

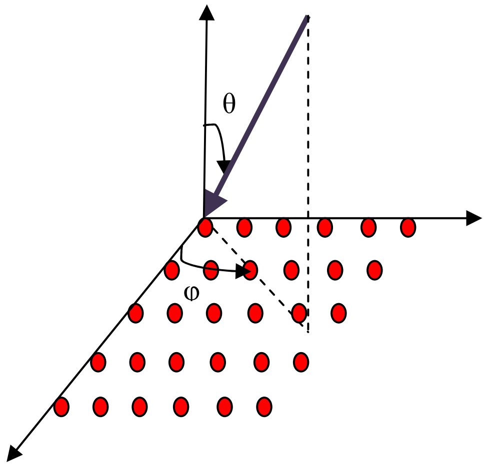

Fig. 4 Rectangular Antenna Array

The resultant array factor is given by:

$$AF(\theta, \emptyset) = \sum_{m=1}^{M}\sum_{n=1}^{N} w_{mn}\, e^{j\left[(m-1)\left(kd_x \sin\llbracket(\theta)\cos(\emptyset)+\beta_x\right)+(n-1)\left(kd_y \sin\llbracket(\theta)\sin(\emptyset)+\beta_y\right)\rrbracket\rrbracket\right]} \qquad (5)$$

Where:AF: Array Factor, k: wave number, d: element separation distance, θ, ∅: wave front angle of arrival, β: phase difference between adjacent elements, w: element amplitude.

The total radiation pattern is the product of individual element radiation pattern and the array factor.[3].

V. METHODOLOGY

A. Null steering

The general idea is to ensure radiation main lobe orientation in desired directions and minimal radiation in undesired directions. As such, the problem is formulated as a maximization problem of the relationship defined in Equation 3 below.

$$\text{Objective function} = \text{AF (desired directions)} - \text{AF (undesired directions)} \qquad (6)$$

AF denotes Array Factor.

In this format, this objective function is easily maximized using the PSO algorithm. The phase weighting in each and every array path is taken as the function variable.

B. Reference signal based beamforming

The general idea is to minimize the MMSE function as described previously.

The MMSE objective function is easily maximized using the PSO algorithm [6]. The phase weighting in each and every array path is taken as the function variable.

VI. RESULTS

Direction set one

The set of angles in Table 1 below is used in the first set of simulations.

Table 1: Set 1 desired signal direction and interferer directions

	Des	Int 1	Int 2	Int 3	Int 4
Elevation angle	30	60	40	10	20
Azimuth angle	300	100	200	130	300

Interferer 1 is intentionally highly correlated to the desired signal to mimic multipath propagation.

A. *Simulation results using 16 elements*

In Fig. 5 is the array response contour plot and a 3-dimension plot emanating from weights derived in the MMSE based beamforming process using 16 elements.

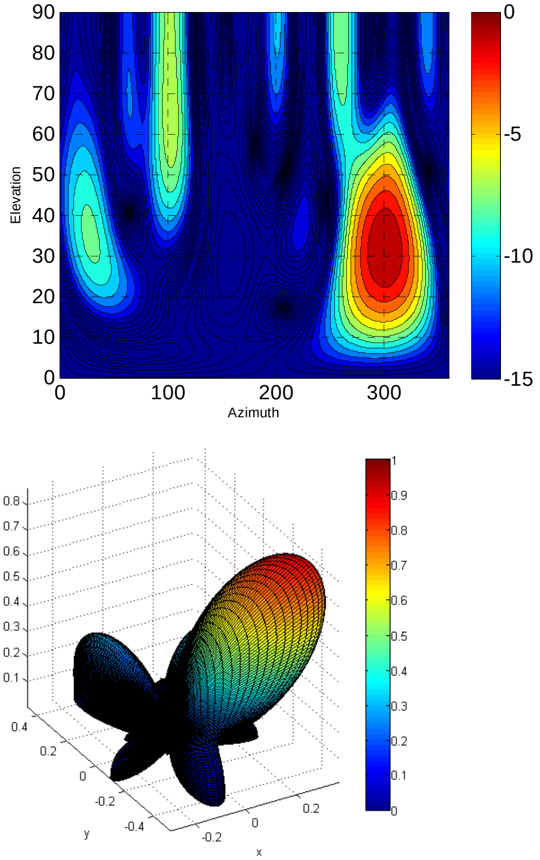

Fig. 5(a) 16 elements set 1 contour plot in dB and 5(b) 3 dimension plot.

It is worthwhile noting that high array response levels are directed towards the direction (30,300). This is the direction of arrival associated with the desired signal. This is despite the fact that no direction information is supplied to the MMSE based beamformer. A relatively high response level is directed towards the direction (60, 100). This is the direction of arrival associated with interferer 1 (simulated to have a high correlation magnitude with the desired signal to mimic multipath propagation). Minimal array response is directed towards directions (40,200) and (10,130). These are the directions of arrival associated with interferers 2 and 3 (uncorrelated with the desired signal). Despite interferer 4 having low correlation with the desired signal, quite a high array response is directed towards its direction of arrival (20,300). This is due to its source being close to that of the desired signal. The 4x4 element array is unable to resolve response levels to suitable values at this degree of closeness.

In Fig. 6 are signal plots emanating from weights derived in the MMSE and null steering based beamforming processes. A visual inspection of these plots shows better performance in the MMSE based beamforming process. Notable differences are found in correlation values tabulated in Table 2 *(correlation D & BF).*

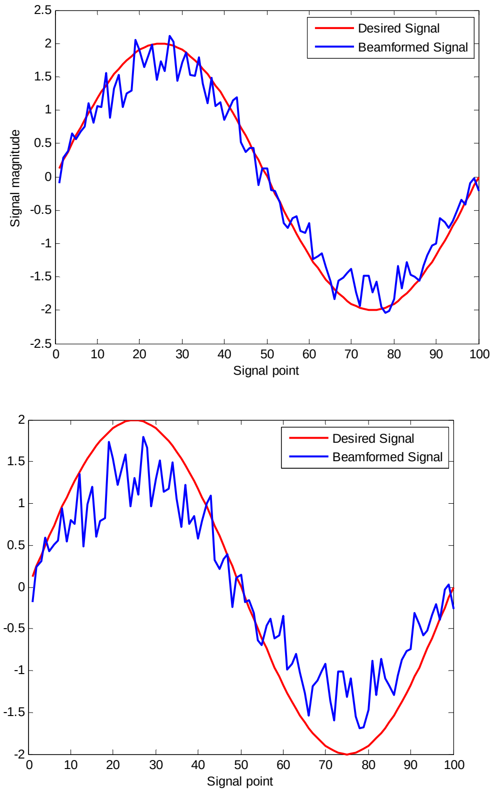

Fig. 6(a) 16 elements set 1 MMSE BF signal plot and 6(b) Null Steering BF signal plot

B. *Simulation results using 64 elements*

Illustrated in Fig. 7 is the array response contour plot and a 3-dimension plot emanating from weights derived in the MMSE based beamforming process using 64 elements.

Again, it is worthwhile noting that high array response levels are directed towards the desired signal direction (30,300): this response is sharper (has a narrower beamwidth) compared to that associated with the 16 element solution. A relatively high response level is directed towards the interferer 1 direction (60, 100), but sharper than that associated with the 16 element solution. Lower response is directed towards interferer 4 (20,300) compared to that associated with the 16 element solution.

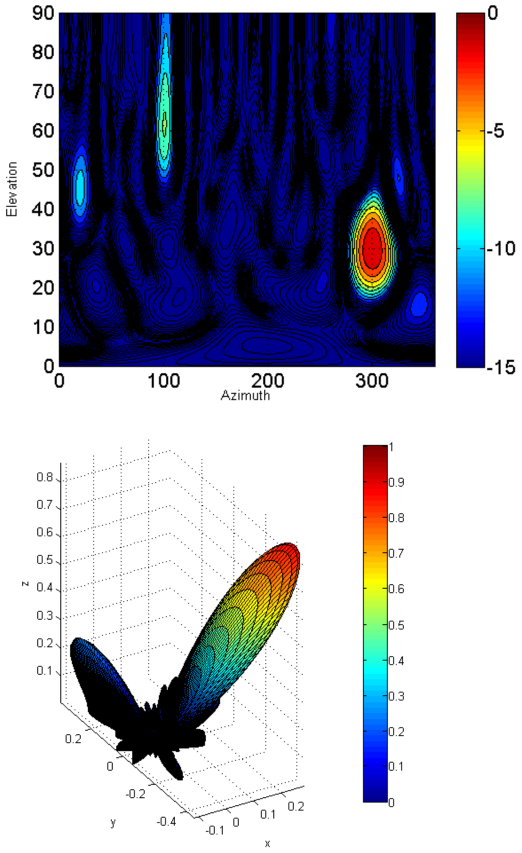

Fig. 7(a) 64 elements set 1 contour plot in dB and 7(b) 3 dimension plot

In Fig. 8 are signal plots emanating from weights derived in the MMSE based and null steering based beamforming processes. A visual inspection of these plots shows better performance in the MMSE based beamforming process. Notable differences are found in correlation values tabulated in Table 2 *(correlation D & BF)*. Higher correlation values imply better performance.

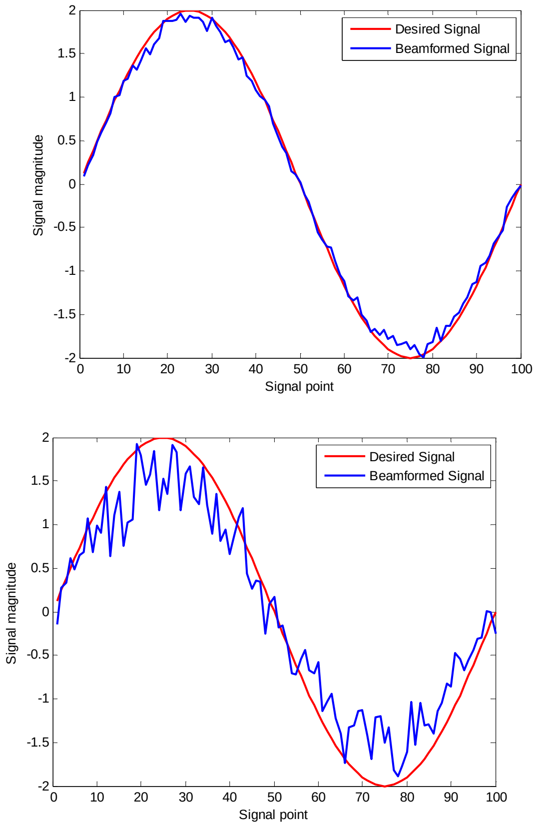

Fig. 8(a) 64 elements set 1 MMSE BF signal plot and 8(b) Null Steering BF signal plot

Table 2 clearly tabulates results obtained (MMSE/ Null steering approaches comparison).

Table 2: Set 1 results

ARRAY RESPONSE MAGNITUDE IN DECIBELS				
	MMSE APPROACH		*NULL STEERING APPROACH*	
DIRECTION	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*
DESIRED	-0.0372	-0.0051	-0.00040528	-0.0044
INTERFERER 1	-6.2531	-7.063	-18.1408	-23.5435
INTERFERER 2	-20.6652	-27.0454	-17.8796	-26.3229
INTERFERER 3	-20.4656	-32.2363	-22.3223	-22.5657
INTERFERER 4	-1.7762	-5.8589	-1.4123	-6.3526
TOTAL INTERFERENCE LEVEL	**-49.1601**	**-72.2036**	**-59.755**	**-78.7847**
OTHER STATISTICAL PARAMETERS IN DECIBELS				
	MMSE APPROACH		*NULL STEERING APPROACH*	
PARAMETER	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*
MAXIMUM RESPONSE	0	0	0	0
MINIMUM RESPONSE	-57.7999	-72.4777	-92.5809	-99.6897
MEAN	-14.6806	-24.3111	-22.6363	-27.8246
MEDIAN	-13.4556	-23.8489	-19.1551	-28.1068
CORRELATION D& I1	-1			
CORRELATION D& I2	0.1719			
CORRELATION D& I3	0.0286			
CORRELATION D& I4	0.1819			
CORRELATION D& BF	0.9095	0.9692	0.8731	0.8972

Direction set two

The set of angles in Table 3 is used in the second set of simulations.

Table 3: Set 2 desired signal direction and interferer directions

	Des	Int 1	Int 2	Int 3	Int 4
Elevation angle	45	40	100	100	35
Azimuth angle	45	10	250	120	45

Interferer 1 is intentionally highly correlated to the desired signal to mimic multipath propagation.

A. *Simulation results using 16 elements*

Illustrated in Fig. 9 is the array response contour plot and a 3-dimension plot emanating from weights derived in the MMSE based beamforming process using 16 elements. Again, it is worthwhile noting that high array response levels are directed towards the desired signal direction (45, 45). A relatively high response level is directed towards the interferer 1 direction (40, 10) owing to its high correlation with the desired signal. A high response level is also directed towards interferer 4 (35, 45) owing to the inability of the 16 element array to resolve the response to a low level in the source region close to that of the desired signal.

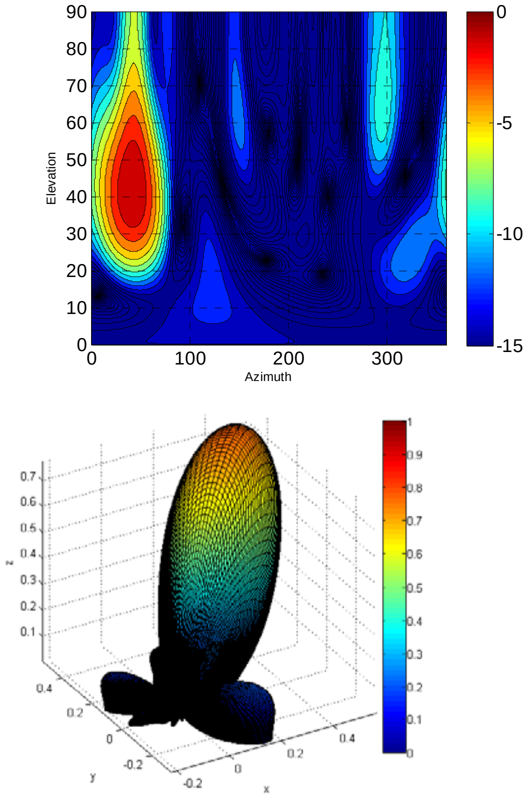

Fig. 9(a) 16 elements set 2 contour plot in dB and 9(b) 3 dimension plot

In Fig. 10 are signal plots emanating from weights derived in the MMSE based and null steering based beamforming processes. A visual inspection of these plots shows better performance in the MMSE based beamforming process. Notable differences are found in correlation values tabulated in Table 4 *(correlation D & BF)*.

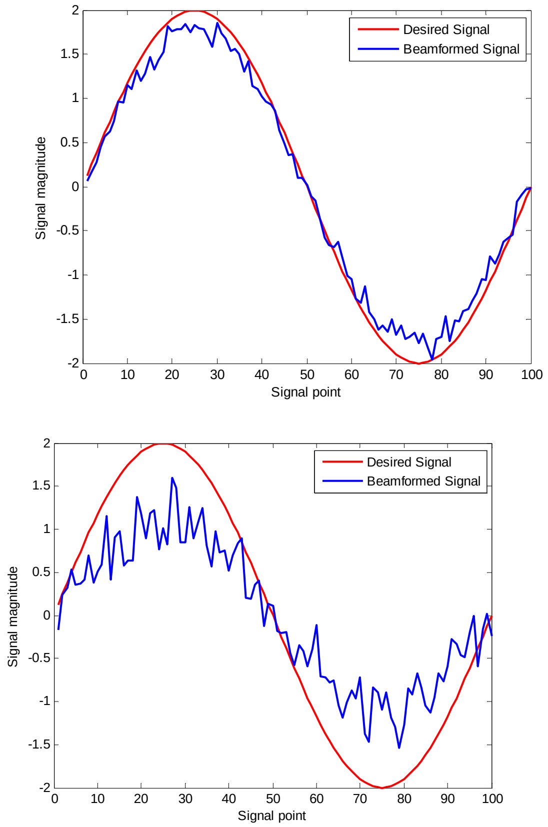

Fig.10(a) 16 elements set 2 MMSE BF signal plot and 10(b) Null Steering BF signal plot

B. *Simulation results using 64 elements*

Illustrated in Fig. 11 is the array response contour plot and a 3-dimension plot emanating from weights derived in the MMSE based beamforming process using 64 elements. Again, it is worthwhile noting that high array response levels are directed towards the desired signal direction (45, 45). This response is shaper (has a narrower beamwidth) compared to that associated with the 16 element solution. A relatively high response level is directed towards the interferer 1 direction (40, 10), but sharper than that associated with the 16 element solution. Lower response is directed towards interferer 4 (35, 45) compared to that associated with the 16 element solution.

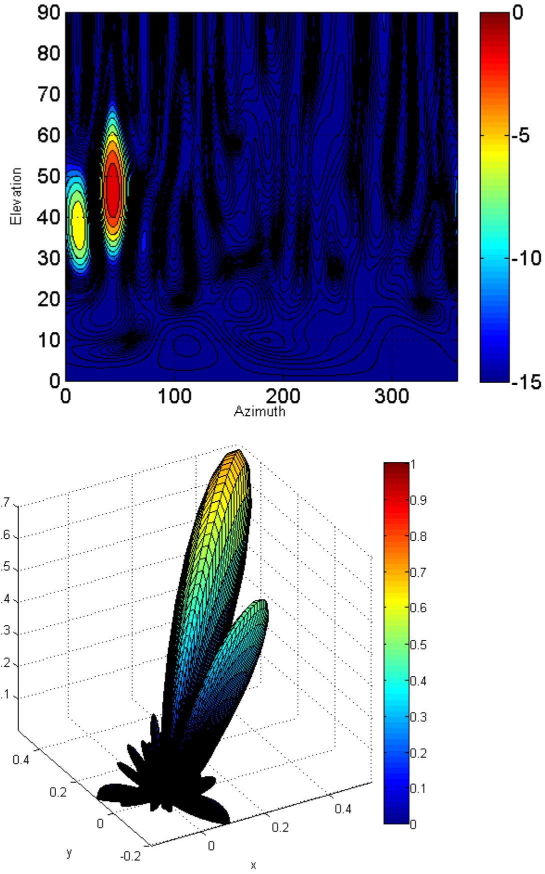

Fig. 11(a) 64 elements set 2 contour plot in dB and 11(b) 3 dimension plot

A visual inspection of the signal plots in Fig. 12 shows better performance in the MMSE based beamforming process.

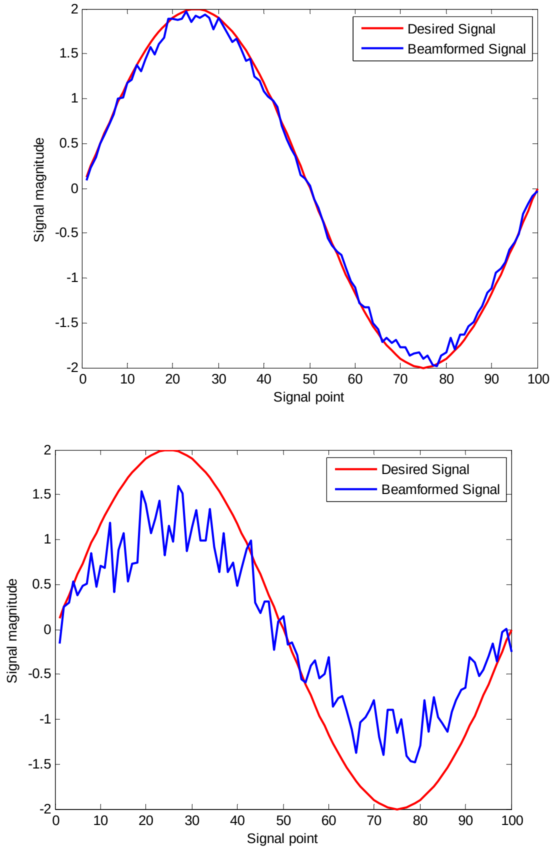

Fig. 12(a) 64 elements set 2 MMSE BF signal plot and 12(b) Null Steering BF signal plot

Table 4: Set 2 results (MMSE/ Null steering approaches comparison)

ARRAY RESPONSE MAGNITUDE IN DECIBELS				
	MMSE APPROACH		*NULL STEERING APPROACH*	
DIRECTION	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*
DESIRED	-0.134	-0.1119	-0.0106	-0.00058297
INTERFERER 1	-4.841	-4.7414	-8.5432	-11.9145
INTERFERER 2	-24.3275	-31.2238	-23.6239	-34.2079
INTERFERER 3	-26.1215	-28.4913	-29.811	-27.0084
INTERFERER 4	-0.5901	-4.9331	-1.1012	-4.0591
TOTAL INTERFERENCE LEVEL	**-55.8801**	**-69.3896**	**-63.0793**	**-77.1899**
OTHER STATISTICAL PARAMETERS IN DECIBELS				
	MMSE APPROACH		*NULL STEERING APPROACH*	
PARAMETER	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*	*16 ARRAY ELEMENTS*	*64 ARRAY ELEMENTS*
MAXIMUM RESPONSE	0	0	0	0
MINIMUM RESPONSE	-65.2077	-74.1498	-90.8245	-76.5478
MEAN	-16.3616	-23.9006	-25.1806	-28.2551
MEDIAN	-15.2612	-23.9076	-21.2897	-28.6457
CORRELATION D& I1	-1			
CORRELATION D& I2	0.1719			
CORRELATION D& I3	0.0286			
CORRELATION D& I4	0.1819			
CORRELATION D& BF	0.924	0.9795	0.833	0.8576

VII. DISCUSSION

In Tables 2 and 4, the maximum response depicts the highest array response level achieved. The minimum response depicts the lowest array response level achieved. The response mean depicts the average array response achieved. The response mean alongside the response median gives an indication of overall side-lobe contributions.

Maximal reception is directed towards the desired direction and minimal reception towards the undesired directions as depicted by the contour plots and 3 dimension plots. The MMSE approach performs better than the null steering approach as depicted by the values of correlation between desired signal and beamformed signal (*correlation d& bf*) as tabulated in Tables 2 and 4; and signal plots as per Figs. 6, 8, 10 and 12. It is worthwhile to note that some reception is focused towards interferer 1, which is highly correlated (a correlation magnitude of 1) with the source signal, in MMSE based beamforming. This signal is intentionally designed to mimic multipath propagation. MMSE based beamforming takes advantage of multipath propagation through phase correction in the beamforming process. Interferer 1 is treated like any other interferer in null steering based beamforming.

A. Isotropic antenna receiver performance

The performance of an isotropic antenna receiver in a noisy and multipath environment is hereby described.

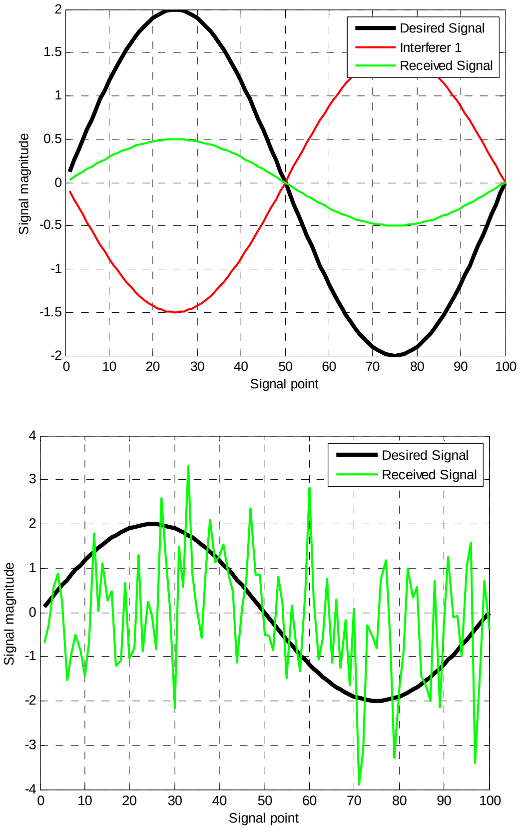

Fig. 13(a) Interferer 1 reception and 13(b) Isotropic reception of all interferers

Fig. 13(a) illustrates the received signal in the presence of a multipath signal (out of phase by 180 degrees). The received signal is a faded replica of the desired signal.

Fig. 13(b) illustrates the received signal in the presence of some four interferences. The correlation between the desired signal and the received signal (in Fig. 9(b)) is 0.364. This value is much lower than that obtained using beamformed array receivers (correlation values tabulated in Tables 2 and 4).

B. Performance of the PSO algorithm

The PSO algorithm performance in MMSE beamforming is superb. An 8x8 antenna array presents a difficult problem in that the presented dimension of the problem is high (64). This coupled with the highly multimodal nature of the problem translates into a difficult optimization problem by all standards. The PSO algorithm did not fail to give optimal weights in all simulations carried out.

VIII. CONCLUSION

The reference signal based beamforming approach is found better than the null steering based approach on the basis of the utilized performance measures, especially in situations encompassing multipath propagation.

REFERENCES

[1] C. A. Balanis, *Antenna Theory, Analysis and Design*, 3rd ed. Hoboken, New Jersey: John Wiley & Sons, Inc., 2005.

[2] A. E. Zooghby, *Smart Antenna Engineering*, 1st ed. Norwood, United States of America: Artech House, 2005.

[3] C. A. Balanis, "Smart Antennas," in *Antenna Theory Analysis and Design*. New Jersey, United States of America: John Wiley & Sons, Inc., Hoboken, New Jersey, 2005, ch. 16, pp. 945-999.

[4] I. Network World. (2010) Networkworld. [Online]. http://networkworld.com/news/2010/101910-smart-antennas-wifi-performance.html

[5] L. C. Godara, *Smart Antennas*, 1st ed. Florida, United States of America, 2004.

[6] T. Weise, *Global Optimization Algorithms: Theory and Applications*. 2009.