Proceedings of the Sustainable Research and Innovation Conference,
JKUAT Main Campus, Kenya
3 - 5 May 2017

Profit Based Unit Commitment in Deregulated Electricity Markets Using A Hybrid Lagrangian Relaxation - Particle Swarm Optimization Approach

Adline K. Bikeri, Christopher M. Maina and Peter K. Kihato

Abstract—In deregulated electricity markets, individual generation companies (GENCOs) carry out independent unit commitment based on predicted energy and revenue prices. The GENCOs unit commitment strategies are developed with the aim of maximizing profit based on the cost characteristics of their generators and revenues from predicted prices of energy and reserve subject to all prevailing constraints in what is known as Profit Based Unit Commitment (PBUC). A tool for carrying out PBUC is an important need for the GENCOs. This paper demonstrates the development of a solution methodology for the PBUC optimization problem in deregulated electricity markets. A hybrid of the Lagrangian Relaxation (LR) and Particle Swarm Optimization (PSO) algorithms is used to determine an optimal UC schedule in a day-ahead market using the expected energy and reserve prices taking advantage of the strengths of both algorithms. The PSO algorithm is used to update the Lagrange multipliers giving a better quality solution. An analysis of the PSO algorithm parameters is carried out to determine the parameters that give the best solution. The algorithm is implemented in MATLAB software and tested for a GENCO with 54 thermal units adapted from the standard IEEE 118-bus test system.

Keywords—Deregulated Electricity Market, Lagrangian Relaxation, Particle Swarm Optimization, Profit Based Unit Commitment.

I. INTRODUCTION

Over the last few decades the electric energy sub-sector has been undergoing significant changes. Probably the biggest change has been deregulation of many power systems especially in the developed world; though aspects of deregulation are also beginning to take root in developing nations. Deregulation refers to the unbundling of vertically integrated power systems into Generation Companies (GENCOs), Transmission Companies (TRANSCOs) and Distribution Companies (DISCOs) [1]. The main aim of deregulation is to create competition among GENCOs and hence provide different choices of generation options at lower prices to consumers [1], [2].

Unit Commitment has always been a significant optimization task in power systems [3], [4]. However, the approach in the deregulated environment is significantly different form that in the regulated environment. Here, the GENCO is not the system operator. This means that, unlike the regulated market where the objective of the utility in unit commitment is the minimization of operating cost, in the deregulated environment, the objective of the GENCO is the maximization of profit. This has led to what is now referred to as Profit Based Unit Commitment (PBUC) in deregulated markets [5].

Numerous methodologies for solving both the traditional UC and PBUC problems have been proposed in literature. These methodologies can be classified as classical methods and non-classical methods. Classical methods include Priority Listing, Dynamic Programming, Branch and Bound, Mixed Integer Programming, and Lagrangian Relaxation (LR) [5], [6]. Non-classical methods include Genetic Algorithms, Particle Swarm Optimization, Artificial Bee Colony, Muller method among others [7], [8]. There have also been proposals for hybridization of some of these methods taking advantage of the strengths of two or more methods to provide a more effective solution algorithm [9]–[11]. A comprehensive review of these methods can be found in [3], [4], [12]

Despite the numerous efforts to solve what is a very complex optimization problem over the past few years, a number of research gaps still exist [4]. This paper formulates the PBUC problem incorporating reserve payments and as well as spot market energy prices. A solution methodology for the PBUC optimization problem is then developed. Here, a hybrid of the Lagrangian Relaxation (LR) and Particle Swarm Optimization (PSO) algorithms is used to determine an optimal UC schedule including constraints of having to meet bilaterally agreed energy supply commitments.

Lagrangian Relaxation (LR) is chosen since is currently the most commonly used approach in the solution of the UC problem. However, several methods for updating the Lagrange multipliers have been proposed. The Particle Swarm Optimization (PSO) algorithm is one such method and is implemented in this paper. The biggest challenge with the PSO algorithm lies in the proper selection of the various weighting factors that largely determine the algorithm's performance. Thus, apart from just implementation of the PSO algorithm, parameters selection is also addressed in this paper.

The rest of the paper is organized as follows: Section II introduces the LR and PSO procedures and their application in the solution of optimization problems. Section III outlines the PBUC problem formulation while Section IV explains the proposed solution methodology. In Section V, simulation results on a test IEEE system are presented including section of PSO parameters, analysis of the obtained optimal solution, algorithm convergence performance, and computation time. Finally, paper conclusions are given in Section VI.

A. K. Bikeri, School of Electrical, Electronic, and Information Engineering, JKUAT (phone: +817040882005; e-mail: adlinebikeri@gmail.com).

C. M. Maina, Department of Electrical and Power Engineering, TUK (e-mail: cmainamuriithi@gmail.com).

P. K. Kihato, School of Electrical, Electronic, and Information Engineering, JKUAT (e-mail: pkihato@jkuat.ac.ke).

II. LAGRANGIAN RELAXATION AND PARTICLE SWARM OPTIMIZATION

The Lagrangian relaxation (LR) method for solving an optimization problem works by incorporating complicated constraints of the problem into the objective function using penalty terms known as Lagrange multipliers [13]. The Lagrange multipliers penalize violations of the corresponding constraints and by systematically updating these penalty factors, an optimal solution of the original problem can be determined. In practice, the modified (relaxed) optimization problem is usually simpler to solve than the original problem hence the application of the method.

The quality of the solution obtained via LR strongly depends on the algorithm used to update the Lagrangian multipliers. Traditionally, gradient based methods have been used but more recently, one or more of the heuristic methods have been applied in an effort to improve the quality of the solution [13], [14].

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique simulating the natural animal's behavior to adapt to the best of the characters among entire populations like bird flocking and fish schooling [15]. Since it's inception in the mid 90's, PSO has been widely applied by researchers in various optimization applications including the solution of the UC problem [7]. In simple terms, a population (swarm) of processing elements called particles, each of which representing a candidate solution forms the basis of computation in the PSO algorithm. A possible solution to the existing optimization problem is represented by each particle in the swarm. A population of random solutions is used to initialize the PSO algorithm and optima are searched by updating the solution in each iteration (epoch).

During a PSO iteration, every particle moves towards its own personal best solution that it achieved so far ($pBest$), as well as towards the global best ($gBest$) solution which is best among the best solutions achieved so far by all particles present in the population. This is done in a random manner ensuring that the algorithm throughly searches the solution space. After a certain pre-set number of iterations (generations), the particle with the global best solution is stored as the optimal solution to the optimization problem.

III. PBUC PROBLEM FORMULATION

The PBUC problem is formulated as as a maximization of a GENCO's profit by deciding an optimal unit commitment schedule based on expected energy and reserve prices. The GENCO's bilateral contract commitments are also considered. The objective function and the operational constraints are given in the following subsections. The variables in the various equations are shown in Table I.

TABLE I
NOMENCLATURE

h	hour index
i	generator index
j	PSO particle index
k	iteration number index
H	number of scheduling hours
J	number of PSO particles
K	maximum number of PSO algorithm generations
N	total number of generators
PF	GENCO Profit
RV	GENCO Revenue
RV_p^h	revenue from energy sales (MWh) at hour h
RV_r^h	revenue from reserve sales at hour h
TC	GENCO Costs
FC_i^h	fuel cost of generator i at hour h
SC_i^h	start up cost of generator i at hour h
a_i, b_i, c_i	constant for fuel cost curve of generator i
γ_i	start up cost of generator i
α_s^h	unit price for spot market energy sales at hour h
α_b^h	unit price for bilateral contracts energy sales at hour h
α_r^h	unit price for reserve capacity sales at hour h
P_b^h	power supply for bilateral contracts at hour h
P_i^h	power output from generator i at hour h
P_i^{min}, P_i^{max}	minimum and maximum outputs of generator i respectively
RU_i, RD_i	ramp up and ramp down limits of generator i respectively
κ	factor for contract of differences
U_i^h	state of generator i at hour h
$\lambda_{j,k}^h$	Lagrange Multiplier for particle j at hour h for iteration k
$\Lambda_{j,k}$	Set of Lagrange Multipliers for particle j at iteration k
$v_{j,k}^h$	velocity of particle j at hour h for iteration k
$V_{j,k}$	Set of velocities for particle j at iteration k
$pBest_j$	Personal best solution of particle j
$gBest$	Global best solution for all particles
w_1, w_2, w_3	weighting factors corresponding to the particle's previous velocity, personal best position and global best position respectively
r_1, r_2	random numbers in $[0 \quad 1]$

A. Objective Function

Profit (PF) is defined as the difference between revenue (RV) obtained from sale of energy and reserve* and the total operating cost (TC) of the GENCO. The objective function of the PBUC problem is then given as:

$$\text{Maximize } PF = RV - TC \tag{1}$$

1) GENCO Revenue: RV is given by:

$$RV = \sum_{h=1}^{H} \left(RV_p^h + RV_r^h \right) \tag{2}$$

Revenue from the energy market at a given hour RV_p^h is calculated as:

$$RV_p^h = \alpha_b^h P_b^h + \alpha_s^h \left(\sum_{i=1}^{N} P_i^h - P_b^h \right) + \kappa \left(\alpha_s^h - \alpha_b^h \right) P_b^h \tag{3}$$

The first term in (3) represents revenue from bilateral contracts, the second term represents revenue from the energy sold at the spot market, while the third term represents revenue from contracts of differences.

Contracts of differences (cfds) are usually included in bilateral contracts to compensate suppliers and consumers

*Revenue from other ancillary services could be included in a similar manner.

for differences between the bilaterally agreed prices and the prevailing market price. A cfd factor of $\kappa = 0$ would mean that the GENCO sells power in the bilateral market at the bilaterally agreed price even if the market price is higher (no compensation) while a cfd factor of $\kappa = 1$ essentially means that the GENCO sells power in the bilateral market at the prevailing market price (full compensation). A value of $\kappa = 0.5$ is adopted in this paper.

Revenue from sale of reserve at hour h is given by:

$$RV_r^h = \alpha_r^h \sum_{i=1}^{N} \left(P_i^{max} - P_i^h\right) \tag{4}$$

2) GENCO Costs: TC is a sum of fuel costs (FC) and start up costs (SC) for all generators over the entire scheduling period. This is given as:

$$TC = \sum_{h=1}^{H}\sum_{i=1}^{N} \left(FC_i^h + SC_i^h\right) \tag{5}$$

where

$$FC_i^h = a_i + b_i P_i^h + c_i \left(P_i^h\right)^2 \tag{6}$$

$$SC_i^h = \gamma_i \left(1 - U_i^{h-1}\right) U_i^h \tag{7}$$

B. Operational Constraints

GENCO operational constraints are given as:

(a) Power balance for bilateral contracts

$$\sum_{i=1}^{N} P_i^h \geq P_b^h \qquad \forall h \tag{8}$$

(b) Generation limit constraints

$$U_i^h P_i^{min} \leq U_i^h P_i^h \leq U_i^h P_i^{max} \qquad \forall i, \forall h \tag{9}$$

(c) Ramp up constraints

$$P_i^h - P_i^{h-1} \leq RU_i \qquad \forall i, \forall h \tag{10}$$

(d) Ramp down constraints

$$P_i^{h-1} - P_i^h \leq RD_i \qquad \forall i, \forall h \tag{11}$$

(e) Minimum up time

$$U_i^h = 1 \quad \text{if } U_i^t - U_i^{t-1} = 1, \text{ for } h = t, ..., t + MUT - 1 \tag{12}$$

(f) Minimum down time

$$U_i^h = 0 \quad \text{if } U_i^{t-1} - U_i^t = 1, \text{ for } h = t, ..., t + MDT - 1 \tag{13}$$

Constraints (9)-(13) are similar to the traditional UC formulation [3]. However, constraint (8) indicates that the GENCO's total generation must be greater than its bilateral contracts commitments. This is in contrast with the traditional case where generation must equal total system demand and losses. Unlike the traditional UC formulation, there is no spinning reserve constraint as this is not the GENCO's responsibility. The GENCO only gets payments for supplying part of the reserve. Revenue from reserve sales is therefore added to the objective function.

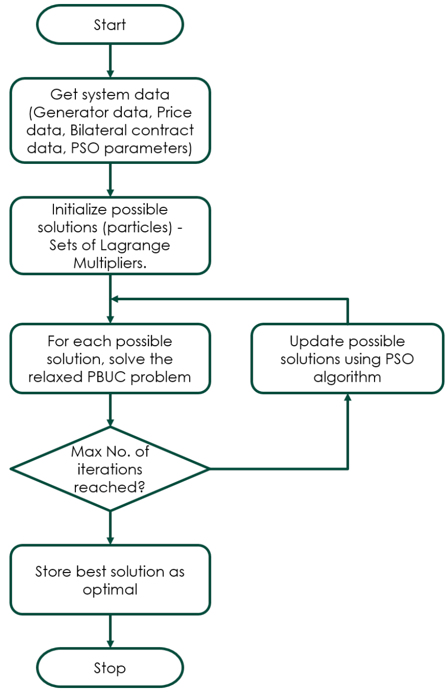

Fig. 1. PBUC solution algorithm using LR-PSO

IV. SOLUTION METHODOLOGY

A. PBUC Solution Algorithm

The basic structure of the solution algorithm for solving the PBUC problem using LR and PSO is shown in Fig. 1. Basically, a Lagrangian function is formed by relaxing constraint (8) into the objective function. This is because it is the only constraint that couples the units. Possible solutions to the relaxed problem are then randomly generated and iteratively solved using a two-step process.

The first step involves solving the relaxed problem for each possible solution (sets of Lagrange multipliers). With the relaxation, optimal schedules of individual generation units can be easily determined by breaking down the relaxed function into subproblems for each unit. A 2-state dynamic programming code is implemented to find an optimal UC schedule for each unit given a set of Lagrange multipliers.

The second step involves updating of the possible solutions (particles) using the PSO algorithm. This is done iteratively for a number of pre-set iterations (maximum number of PSO generations). The two steps are outlined in the following subsections.

B. Solution of the Relaxed Problem

Constraint (8) – the power balance for bilateral contracts – is the only constraint that couples the generating units and is therefore relaxed by being included in the objective function to form the Lagrangian function L as:

$$L = RV - TC - \sum_{h=1}^{H} \lambda^h \left(P_b^h - \sum_{i=1}^{N} P_i^h\right) \tag{14}$$

The relaxed problem is therefore the maximization of L subject to constraints (9) to (13).

To maximize L with respect to P_i^h in (14):

$$\frac{\partial L}{\partial P_i^h} = 0 \quad \forall i, h \tag{15}$$

i.e.

$$\frac{\partial L}{\partial P_i^h} = \left(\alpha_s^h - \alpha_r^h\right) - \left(b_i + 2c_i P_i^h\right) + \lambda^h = 0 \tag{16}$$

hence

$$P_i^h = \frac{\alpha_s^h - \alpha_r^h + \lambda^h - b_i}{2c_i} \tag{17}$$

The following procedure is thus used to solve the relaxed PBUC problem for a set of Lagrange multipliers: $\Lambda = \{\lambda^1, \lambda^2, \ldots, \lambda^H\}$.

Step 1: Get input data (generator cost data, hourly price data, Lagrangian multipliers)
Step 2: Set $i = 1$
Step 3: Set $h = 1$
Step 4: calculate P_i^h from (17)
Step 5: check for generator limit constraints
if $P_i^h > P_i^{max}$ set $P_i^h = P_i^{max}$
if $P_i^h < P_i^{min}$ set $P_i^h = P_i^{min}$
Step 6: check the ramp up and ramp down constraints and change P_i^h accordingly
Step 7: check the minimum up time and minimum down time constraints and change P_i^h accordingly
Step 8: determine the optimal UC schedule using 2-state dynamic programming
Step 9: $h = h + 1$. If $h \leq H$ go to **Step 4**. Else go to **Step 10**
Step 10: $i = i + 1$. If $i \leq N$ go to **Step 3**. Else go to **Step 11**
Step 11: Calculate total revenue, costs and profits
Step 12: Store the results (UC status for all generators, scheduled power, profit)

C. Lagrange Multipliers Update via Particle Swarm Optimization

The PSO algorithm is used to update the Lagrange Multipliers to determine the set that provides the best results. A particle represents a candidate solution which is a set of Lagrange Multipliers – one for each hour of the scheduling horizon. For a scheduling period of H hours, the j^{th} particle after k iterations $\Lambda_{j,k} = \{\lambda_{j,k}^1, \lambda_{j,k}^2, \lambda_{j,k}^3, \ldots, \lambda_{j,k}^H\}$ represents a position in the H-dimension solution space. The particle also has an associated velocity $V_{j,k} = \{v_{j,k}^1, v_{j,k}^2, v_{j,k}^3, \ldots, v_{j,k}^H\}$ which represents a direction in which the particle is moving in the solution space.

The PSO algorithm moves the particles around the solution space after each iteration in a search for the best possible solution. The particle position update follows two "best" positions: $pBest$ and $gBest$. $pBest_j$ is the j^{th} particle's personal best solution found so far while $gBest$ is the entire population's global best solution (the best amongst the various $pBest$s).

At each iteration, the velocity of each particle is updated using†

$$V_{j,k+1} = w_1 V_{j,k} + w_2 r_1 \left(pBest_j - \Lambda_{j,k}\right) + w_3 r_2 \left(gBest - \Lambda_{j,k}\right) \tag{18}$$

†see variable definitions on the nomenclature list in Table I.

The position is then updated using the move equation:

$$\Lambda_{j,k+1} = \Lambda_{j,k} + V_{j,k+1} \tag{19}$$

The following procedure is used to solve the PBUC problem updating candidate solutions (sets of Lagrange Multipliers) using the PSO algorithm:

Step 1: Randomly initialize J particles (candidate solutions)
Step 2: set $k = 1$
Step 3: set $j = 1$
Step 4: Solve the relaxed PBUC problem for the j^{th} particle and determine the corresponding GENCO profit $PF_{j,k}$
Step 5: If $k = 1$, set $pBest_j = PF_{j,k}$
else if $PF_{j,k} > pBest_j$; set $pBest_j = PF_{j,k}$
Step 6: $j = j + 1$.
If $j < J$ go to step 4. Else go to step 7
Step 7: Determine $gBest$ as:
$gbest = \max\{pBest_1, pBest_2, \ldots, pBest_J\}$
Step 8: set $j = 1$
Step 9: Update the velocity of particle j using (18)
Step 10: Update the position of particle j using (19)
Step 11: $j = j + 1$.
If $j < J$ go to **Step 9**. Else go to **Step 12**
Step 12: $k = k + 1$
If $k \leq K$ go to **Step 3**. Else STOP

V. SIMULATION RESULTS

A. Test System

The algorithm is tested for a GENCO with 54 thermal units. The generator data is adapted from the IEEE 118-bus test system and obtained from http://motor.ece.iit.edu/data/PBUCData.pdf. The GENCO's own load (bilateral market commitment) is assumed to be constant at 3,500 MW with PEAK and OFF-PEAK prices as shown in Table II.

B. Selection of PSO Parameters

The quality of the solution obtained from the PSO algorithm is largely dependent on the values of the parameters used. Parameter selection is done in this paper by trying various combinations of the weighting factors w_1, w_2, and w_3 in (18). w_1 was varied from 0.25 to 1.0 in steps of 0.25 while w_2 was varied from 1.0 to 3.0 in steps of 0.5. w_3 was set using the formula: $w_2 + w_3 = 4$ as suggested in literature [15]. These settings give 20 different combinations of the PSO parameters

TABLE II
PRICE DATA

Hour	Energy Price	Reserve Price	Bilateral Price	Hour	Energy Price	Reserve Price	Bilateral Price
1	29.23	2.00	30.00	13	57.01	2.77	56.00
2	26.40	1.70	30.00	14	54.42	2.87	56.00
3	22.47	1.27	30.00	15	63.12	2.92	56.00
4	21.07	1.12	30.00	16	65.59	3.32	56.00
5	23.16	1.35	30.00	17	67.24	3.23	56.00
6	30.86	2.18	30.00	18	63.87	2.97	56.00
7	31.56	2.17	30.00	19	55.61	2.96	56.00
8	47.39	2.34	56.00	20	52.55	2.73	56.00
9	49.70	2.51	56.00	21	47.55	2.35	30.00
10	52.10	2.69	56.00	22	39.69	1.76	30.00
11	55.35	2.94	56.00	23	37.00	1.57	30.00
12	55.50	2.95	56.00	24	30.51	1.16	30.00

as shown in table III. In each case, the number of particles was set to $J = 20$ and the number of PSO iterations was set to $K = 500$. The Lagrange multipliers were initialized to take random values ranging from 0 to 50. The velocity was however not restricted so that the final value of the Lagrange multipliers could be any positive real number. For each combination of PSO parameters, 10 different trials of the PSO algorithm were run and the solutions analyzed.

The maximum profit, average profit, and minimum profit from each combination of PSO parameters was determined and the results are shown in Fig. 2. From Fig. 2, it is seen that the 12^{th} combination of PSO parameters ($w_1 = 0.75$; $w_2 = 1.5$; and $w_3 = 2.5$) provides the best results. Hence, for the simulations in this paper these values are chosen as the PSO parameters.

C. Optimal Solution

1) Unit Commitment: The Unit Commitment schedule for the best solution amongst all the trials carried out is shown in Fig. 5. The horizontal axis represents the scheduling hour while the vertical axis refers to the unit number. A single box in the grid therefore indicates whether a unit is ON (shown in red) or OFF (shown in white). The results show that some of the units e.g. 27 and 45 are ON throughout the day while others such as 33 and 46 are OFF throughout the day. Most of the units are ON or OFF depending on the market price at a given hour.

2) Optimal Power Schedule: Fig. 3 shows the total committed generation for the 24 hours and the GENCO's own load from the UC schedule of Fig. 5. It also indicates the day's total profit as \$2,355,259. From Fig. 3, the total scheduled power from the LR-PSO algorithm is always greater than the GENCO's load. There is no deficiency in meeting the bilateral contract agreements hence the value of the Energy Not Served (ENS) is indicated as zero. Should there be a deficiency in meeting the total committed schedule, the value of ENS will be greater than zero. The value of $ENS = 0$ is ensured by penalizing a result in which $ENS > 0$ when determining the $pBest$ and $gBest$ value in the PSO algorithm.

3) Optimal Values of Lagrange Multipliers: Fig. 4 shows the resulting values of the Lagrange Multipliers corresponding to the schedule shown in Fig. 5. It is observed that the LMs are larger for durations of low market price (hrs 0 to 8) and when the market price is lower than the bilateral contract price (hr 14, 20, 21). In these cases, it is relatively expensive to participate in the spot market but it is necessary to generate power to meet bilateral contract commitments. During the periods of relatively high spot market price and when the spot market price is higher than the bilaterally agreed price, constraint (8) is met and there is no need to add a penalty factor hence the value $LM = 0$.

4) Solution Convergence and Computation Time Analysis: Fig. 6 shows the evolution of the best solution (value of $gBest$) as well as the computation time against the algorithm iteration number. It is seen that after about 300 iterations, the optimal solution does not change much hence it is sufficient to say that 500 iterations are enough for the current problem size. The solution time increases linearly with the number of iterations hence increasing the number of iterations would only increase the computation time without significantly improving the best solution.

VI. Conclusion

A solution methodology that combines the Lagrangian relaxation technique with the heuristic particle swarm optimization techniques to solve the profit based unity commitment problem for GENCOs in deregulated markets has been

TABLE III
PSO Parameter Sets

Set No.	w_1	w_2	w_3	Set No.	w_1	w_2	w_3
1	0.25	1.00	3.00	11	0.75	1.00	3.00
2	0.25	1.50	2.50	12	0.75	1.50	2.50
3	0.25	2.00	2.00	13	0.75	2.00	2.00
4	0.25	2.50	1.50	14	0.75	2.50	1.50
5	0.25	3.00	1.00	15	0.75	3.00	1.00
6	0.50	1.00	3.00	16	1.00	1.00	3.00
7	0.50	1.50	2.50	17	1.00	1.50	2.50
8	0.50	2.00	2.00	18	1.00	2.00	2.00
9	0.50	2.50	1.50	19	1.00	2.50	1.50
10	0.50	3.00	1.00	20	1.00	3.00	1.00

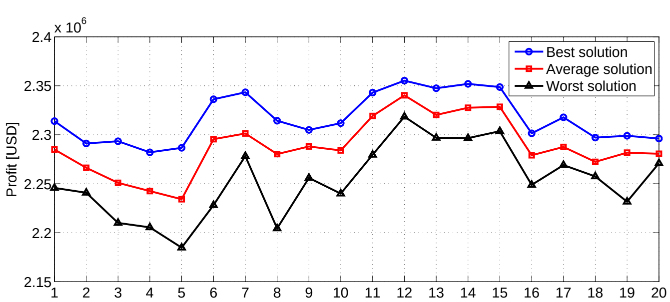

Fig. 2. PSO Parameter Sets Performance

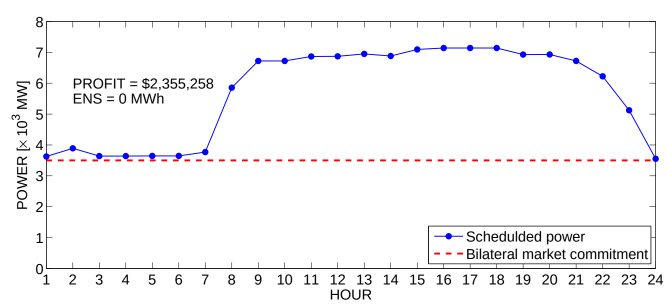

Fig. 3. Optimal GENCO Total Power Generation Schedule

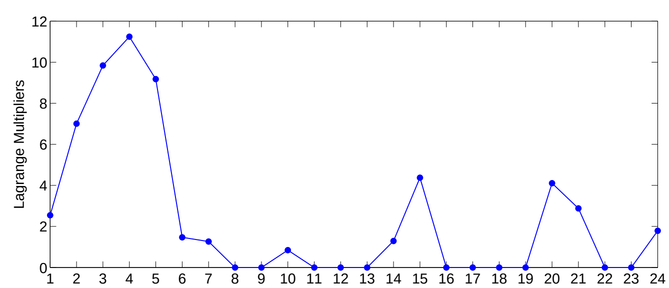

Fig. 4. Lagrange Multipliers Corresponding to the Optimal Solution

Gen No.	HOUR																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
7	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
14	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
15	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
16	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
17	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
19	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
23	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
31	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
35	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
36	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
37	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
38	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
43	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
48	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
51	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
52	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
53	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
54	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0

Fig. 5. Unit Commitment Schedule Corresponding to the Optimal Solution

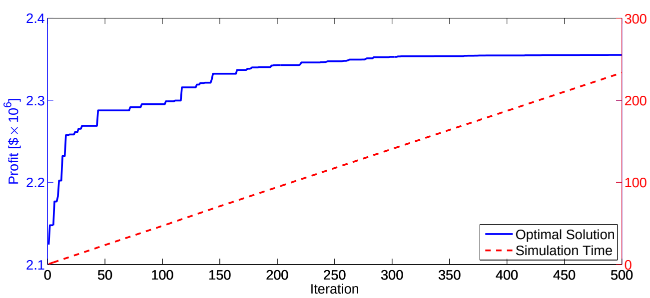

Fig. 6. Analysis of solution convergence and computation time

proposed in this paper. The problem has been formulated including a constraint setting the minimum GENCO output at a given hour as the bilaterally committed generation for the hour. The parameters $w_1 = 0.75, w_2 = 1.5$, and $w_3 = 0.25$ have been chosen based on an assessment of the performance of various combinations of PSO parameters in the solution of the PBUC problem. An implementation for a GENCO with 54 thermal units shows the effectiveness of the proposed methodology.

References

[1] M. Shahidehpour and M. Alomoush, *Restructured Electrical Power Systems, Operation, Trading, and Volatility*. New York: Marcel Decker, 1st ed., 2000.

[2] L. Philipson and H. L. Willis, *Understanding Electric Utilities and De-Regulation*. Florida, USA: Taylor & Francis Group, 2nd ed., 2006.

[3] B. Saravanan, S. Das, S. Sikri, and D. P. Kothari, "A solution to the unit commitment problem — a review," *Frontiers in Energy*, vol. 7, no. 2, pp. 223–236, 2013.

[4] A. K. Bikeri, C. M. Muriithi, and P. K. Kihato, "A review of unit commitment in deregulated electricity markets," in *Scientific Research and Innovation (SRI) Conference, 2015*, pp. 1–5, May 2015.

[5] T. Li and M. Shahidehpour, "Price-based unit commitment: a case of Lagrangian relaxation versus mixed integer programming," *Power Systems, IEEE Transactions on*, vol. 20, pp. 2015–2025, Nov 2005.

[6] A. Rajan, C. Christober, P. Sundarajan, V. Jamuna, R. Madhusubash, and B. Udayakumar, "Multi-area unit commitment in deregulated electricity market using DP approach," *Inter. Journal on Recent Trends in Engineering and Technology*, vol. 3, pp. 210–213, May 2010.

[7] J. Raglend, C. Raghuveer, G. R. Avinash, N. Padhy, and D. Kothari, "Solution to profit based unit commitment problem using particle swarm optimization," *Applied Soft Computing*, vol. 10, no. 4, pp. 1247–1256, Sept. 2010.

[8] H. S. Madraswala and A. S. Deshpande, "Genetic algorithm solution to unit commitment problem," in *2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPE-ICES)*, pp. 1–6, July 2016.

[9] S. Selvi, M. Moses, and C. Rajan, "LR-EP approach for solving profit based unit commitment problem with losses in deregulated markets," *Przeglad Elektrotechniczny*, vol. 11, pp. 210–213, 2013.

[10] K. Lakshmi and S. Vasantharathna, "Hybrid artificial immune system approach for profit based unit commitment problem," *J. Electrical Eng. Technology*, vol. 8, no. 1, pp. 742–751, March 2013.

[11] A. Sudhakar, C. Karri, and A. J. Laxmi, "Profit based unit commitment for {GENCOs} using lagrange relaxationdifferential evolution," *Engineering Science and Technology, an International Journal*, pp. –, 2016.

[12] N. Padhy, "Unit commitment problem under deregulated environment - A review," in *Power Engineering Society General Meeting, 2003, IEEE*, vol. 2, pp. 11088–1094, July 2003.

[13] C. Lemarechal, *Computational Combinatorial Optimization - Optimal or Provably Near-Optimal Solutions*, ch. Lagrangian relaxation, pp. 112–156. Berlin: Springer-Verlag, 2001.

[14] S. Gao, *Bio-Inspired Computational Algorithms and Their Applications*. InTech, 2012.

[15] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Neural Networks, 1995. Proceedings., IEEE International Conference on*, vol. 4, pp. 1942–1948 vol.4, Nov 1995.