Assessment of Voltage Stability Indices for Determination of Power System Weak Buses.

Weldon K. Koskei¹, David K. Murage², Samuel Kangethe³ and Michael J. Saulo⁴

Abstract—The ever growing population and the high level of industrialization globally has led to increase demand for electric power. However, the expansion of electric power generation as well as transmission and distribution power system has been limited by financial constraints. This has led to operation of power system close to its operating stability limits. This overloaded operation condition of power system leads to instability challenges like voltage instability due to inability of the power system to supply required reactive power. In the case that voltage stability status of a stressful system not determined and the challenges arising not addressed, this can lead to voltage collapse and total blackout of the system. Hence there is need to constantly check the status of voltage stability and determine voltage collapse proximity and plan counter corrective measures accordingly. This paper aims to compare Fast Voltage Stability Index (FVSI) and Line stability Index (L_{mn}). These line voltage stability indices indicates the proximity of power system lines to voltage instability. This will help power system planners and operators in their daily power system operations. The standard IEEE 30 bus power system was used to carry out this research and test performance of these voltage stability indices.

Keywords— Voltage Stability, Voltage Collapse, Fast Voltage Stability Index, Line Stability Index.

I. INTRODUCTION

The ever growing demand for electric power due to rising business operations as well as rapid growth of industries has led to operation of power systems closely to their stability limits. These power systems are heavily loaded due to high level of electricity consuming demand which has led to instability challenges [1], [2]. Voltage instability due to insufficient reactive power in the power system is one of the major challenges to power system operators and planners.

Voltage stability is referred to as ability of the power system to maintain steady voltages at all buses in the system under a normal operating conditions and after being subjected to a disturbance [3], [4]. Voltage instability is mainly caused by increase in the load demand and inability of the system to supply required reactive power which leads to uncontrollable voltage drops. Voltage stability is classified into large

Weldon K. Koskei, MSc Student, Department of Electrical and Electronic Engineering, JKUAT, (phone: +254729918244; e-mail: <u>wkoskei@jkuat.ac.ke</u>), David K. Murage, Department of Electrical and Electronic Engineering, JKUAT, (e-mail: <u>dkmurage25@yahoo.com</u>), disturbance and small disturbance voltage stability. Large disturbance voltage stability is concern with maintaining voltages after a large disturbances such as faults and loss of generation in the system while Small disturbance voltage stability deals with maintaining voltages after a small disturbances such as gradual changes of the loads in the system. Voltage stability analysis have gained more interest due to severe voltage instability incidents which occurred in some countries like Japan, Belgium, USA and many parts of the world some decades ago [5], [6].

The information about how close the power system operating point is from voltage collapse is very important to system planners and operators in planning counter corrective measures accordingly. Voltage stability indices indicates the proximity of power system to voltage instability where values close to zero are considered stable but values close to 1 are considered unstable.

Voltage stability analysis using different indices was carried out in this paper and the results obtained from simulating standard IEEE 30 bus were discussed.

II. VOLTAGE STABILITY INDICES

The status of voltage stability in a power system can be determined by using different tools to estimate the closeness of a various operating point to voltage instability condition. Loading margin is one of the tools used and it involve increment of load demand until voltage collapse is reached. It uses PV and QV curves which need to be generated at every bus requiring many calculations and more time. Other voltage stability indices which are used and compared in this paper are discussed below.

A. FAST VOLTAGE STABILITY INDEX

This voltage stability index was developed by Ismail Musirin [7] by considering current equation through a transmission line in two bus system as shown in Fig 1.

Samuel Kangethe, Department of Electrical and Electronic Engineering, JKUAT, (e-mail: <u>samuel.kangethe@jkuat.ac.ke</u>),

Michael J. Saulo, Department of Electrical and Electronic Engineering, Technical University of Mombasa, TUM, (e-mail: <u>michaelsaulo@tum.ac.ke</u>),



Fig 1: Two - Bus transmission line representation Where

 V_1 , V_2 is the voltages at sending and receiving buses

 P_1 , Q_1 is the active and reactive powers at the sending bus

 P_2 , Q_2 is the active and reactive powers at the receiving bus

 S_1 , S_2 is the apparent powers at sending and receiving bus

 δ is the angle difference between sending and receiving bus voltages ($\delta_2 - \delta_1$)

Taking sending bus as reference bus i.e $\delta_1 = 0$ and $\delta_2 = \delta$, the general current equation can be written as

$$I = \frac{v_1 \angle o - v_2 \angle \delta}{R + jX} \tag{1}$$

Where R is the line resistance and X is the line reactance

$$V_2^2 - \left(\frac{R}{x}\sin\delta + \cos\delta\right)V_1V_2 + \left(X + \frac{R^2}{x}\right)Q_2 = 0$$
(2)

The roots for V_2 is calculated as

$$V_2 = \frac{\left(\frac{R}{X}\sin\delta + \cos\delta\right)V_1 \pm \sqrt{\left[\left(\frac{R}{X}\sin\delta + \cos\delta\right)V_1\right)\right]^2 - 4\left(X + \frac{R^2}{X}\right)Q_2}}{2} \quad (3)$$

The real roots of V_2 can be obtained by setting discriminant as greater than or equal to zero as

$$\frac{4Z^2 Q_2 X}{(V_1)^2 (R \sin \delta + X \cos \delta)^2} \le 1 \tag{4}$$

The angle difference is normally very small ($\tilde{\delta} \approx 0$), hence, Rsin $\delta \approx 0$ and X cos $\delta \approx X$

By taking the symbol 'i' as the sending end and 'j' as receiving end, the fast voltage stability index can be expressed as

$$FVSI_{ij} = \frac{4Z_{ij}^2 Q_j}{v_i^2 X_{ij}}$$
(5)

Where

 Z_{ij} - line impedance

 Q_i - receiving end reactive power

 V_i - sending end voltage

X_{ii} - line reactance

If the line index value is close to 0, the line is voltage stable and if it is close to 1, it is close to voltage collapse.

B. LINE STABILITY INDEX

This voltage stability index was developed by Moghavvemi [8] by considering power flow equations through a transmission line in two bus system as shown in Fig 2.

$$V_{s} \angle \delta_{1}, S_{s} = P_{s} + jQ_{s}$$

$$R + jX$$

$$V_{r} \angle \delta_{2}, S_{r} = P_{r} + jQ_{r}$$

Fig 2: One-Line diagram of transmission line

Where

 V_s , V_r is the voltages at sending and receiving buses

 P_s , Q_s is the active and reactive powers at the sending bus

 P_r , Q_r is the active and reactive powers at the receiving bus

 S_s , S_r is the apparent powers at sending and receiving bus

δ is the angle difference between sending and receiving bus voltages ($δ_2 - δ_1$)

By applying power flow concept in the line, the power flow at the sending and receiving end can be formulated as θ - line impedance angle

$$S_s = \frac{|V_s|^2}{z} \mathcal{L}_{\theta} - \frac{|V_s||V_r|}{z} \mathcal{L}_{(\theta + \delta_1 - \delta_2)}$$
(6)

$$S_r = \frac{|V_s||V_r|}{z} \mathcal{L}(\theta - \delta_1 + \delta_2) - \frac{|V_r|^2}{z} \mathcal{L}(\theta - \delta_1 + \delta_2)$$
(7)

Separating active and reactive powers from above equations will give

$$P_r = \frac{v_s v_r}{z} \cos(\theta - \delta_1 + \delta_2) - \frac{v_r^2}{z} \cos\theta$$
(8)

$$Q_r = \frac{v_s v_r}{z} \sin(\theta - \delta_1 + \delta_2) - \frac{v_r^2}{z} \sin\theta$$
(9)

Replacing $\delta_1 - \delta_2 = \delta$ in the above reactive power equation and solving for V_r

$$V_r = \frac{V_s \sin(\theta - \delta) \pm \sqrt{[(V_s \sin(\theta - \delta))]^2 - 4ZQ_r \sin\theta}}{2\sin\theta}$$
(10)

Substituting Z sin θ = X in the above equation

$$V_r = \frac{V_s \sin(\theta - \delta) \pm \sqrt{[(V_s \sin(\theta - \delta))]^2 - 4XQ_r}}{2\sin\theta}$$
(11)

To obtain real roots of V_r , the following conditions must be satisfied

$$\{[(\mathsf{V}_{s}\sin(\theta - \delta))]^2 - 4XQ_r\} \ge 0 \tag{12}$$

$$\frac{4\lambda Q_r}{(V_s \sin(\theta - \delta))]^2} \le 1 \tag{13}$$

By taking the symbol 'i' as the sending end and 'j' as receiving end, the Line stability index can be expressed as

$$L_{mn} = \frac{4X_{ij}Q_j}{[V_i \sin(\theta - \delta)]]^2}$$
(14)

Where

X_{ij} - line reactance

Q_j - receiving end reactive power

- V_i sending end voltage
- $\boldsymbol{\theta}$ line impedance angle
- δ angle between sending end and receiving end voltage

If the line index value is close to 0, the line is voltage stable and if it is close to 1, it is close to voltage collapse.

III. TEST SYSTEM AND SIMULATIONS RESULTS

The voltage stability analysis was performed on standard IEEE 30 bus system. This standard test system is as shown in Fig 3. This system is composed of six generator buses, twenty four load buses and forty one interconnected transmission lines. Static loads were considered in this simulations and the system data is as shown in the appendix.



Fig 3: Standard IEEE 30 Bus Test System

The load flow simulations were carried out using MATLAB software and the results obtained were used to calculate the indices as per the formulas above. Different scenarios were considered to cater for light loading, base loading and overloading conditions and some lines were randomly chosen for analysis. Table I shows calculated indices for base loading. Table II shows the indices obtained for light loading (90% of base loading) and Table III shows indices calculated from results obtained for overloading condition (110% of base loading).

| Table I: | Base | Loading | Results |
|----------|------|---------|---------|
|----------|------|---------|---------|

| From Bus | To Bus | FVSI | L _{mn} |
|----------|--------|--------|-----------------|
| 3 | 4 | 0.0347 | 0.0321 |
| 6 | 9 | 0.0125 | 0.0087 |
| 6 | 10 | 0.0359 | 0.0367 |
| 6 | 28 | 0.0138 | 0.0129 |
| 8 | 28 | 0.0391 | 0.0288 |
| 16 | 17 | 0.0156 | 0.0247 |
| 15 | 18 | 0.0207 | 0.0198 |
| 25 | 26 | 0.2537 | 0.2522 |
| 25 | 27 | 0.0311 | 0.0293 |
| 27 | 28 | 0.1207 | 0.1141 |
| 27 | 29 | 0.2528 | 0.2572 |
| 27 | 30 | 0.2622 | 0.2575 |

The values of the calculated indices in table I above generally ranges between 0.0125 and 0.2622 for both FVSI and L_{mn} . These values are closer to zero indicating that the system is

stable at base loading condition. Generally the values of FVSI are slightly higher than those of L_{mn} for this scenario.

| Table II: Light Loading Results | | | | | | |
|---------------------------------|--------|--------|-----------------|--|--|--|
| From Bus | To Bus | FVSI | L _{mn} | | | |
| 3 | 4 | 0.0287 | 0.0211 | | | |
| 6 | 9 | 0.0118 | 0.0068 | | | |
| 6 | 10 | 0.0318 | 0.0278 | | | |
| 6 | 28 | 0.0104 | 0.0119 | | | |
| 8 | 28 | 0.0245 | 0.0230 | | | |
| 16 | 17 | 0.0142 | 0.0092 | | | |
| 15 | 18 | 0.0187 | 0.0191 | | | |
| 25 | 26 | 0.1491 | 0.1475 | | | |
| 25 | 27 | 0.0226 | 0.0237 | | | |
| 27 | 28 | 0.1182 | 0.1024 | | | |
| 27 | 29 | 0.1512 | 0.1501 | | | |
| 27 | 30 | 0.1594 | 0.1521 | | | |

The results in table II were obtained by reducing both active and reactive power demand of the nominal loads at each bus. These results under light loading conditions further shows the system is stable under this scenario as the values of both FVSI and L_{mn} indices are closer to zero. This shows that the reactive power generated by the system meet the load demand. Just like base loading condition, this scenario shows that the values of FVSI are slightly higher than those of L_{mn} .

Table III: Overloading Condition Results

| | | , , | |
|----------|--------|--------|-----------------|
| From Bus | To Bus | FVSI | L _{mn} |
| 3 | 4 | 0.0621 | 0.0539 |
| 6 | 9 | 0.0493 | 0.0281 |
| 6 | 10 | 0.1028 | 0.0948 |
| 6 | 28 | 0.0812 | 0.0582 |
| 8 | 28 | 0.0901 | 0.0938 |
| 16 | 17 | 0.0625 | 0.0611 |
| 15 | 18 | 0.0862 | 0.0749 |
| 25 | 26 | 0.7122 | 0.7037 |
| 25 | 27 | 0.2404 | 0.2476 |
| 27 | 28 | 0.2870 | 0.3146 |
| 27 | 29 | 0.8331 | 0.8299 |
| 27 | 30 | 0.8791 | 0.8701 |

The results in table III were obtained by increasing both active and reactive power demand of the base loads at each bus. The results from some branches show that the system is stressed under this scenario. The indices values for line 25-26, 27-29 and 27-30 are shown to be closer to one indicating that they are heavily loaded and can cause a voltage collapse of the system. The generators under this operation condition are not capable to supply reactive power required by the loads. Hence the buses connected to these branches are identified as weakest points in the system. The values of both FVSI and L_{mn} show that bus 26,

29 and 30 are weak buses in this system. The highest index is obtained at bus 30 indicating that it is the weakest bus in the whole system.

The results obtained from all simulation scenarios generally show that the values of FVSI are higher as compared to those of L_{mn} . This shows that FVSI index is most sensitive in identifying weakest points in the power system as compared to L_{mn} index.

IV. CONCLUSION

Voltage stability index can be used to identify the weak points in the power system. In this paper, two voltage stability indices were compared under different loading conditions of the system. From the results obtained and analyses, FVSI index was found to be more sensitive in identifying weak areas of a power system as compared to L_{mn} index. Identification of these weak points can help in placement of reactive power compensation systems in the power system to avoid voltage collapse.

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Appendix IEEE 30 BUS SYSTEM DATA

| Bus No | type | Pd | Qd | Vm | Va | Vmax | Vmin |
|-----------|------|------|------|-------|--------|------|------|
| 1 | 3 | 0 | 0 | 1.06 | 0 | 1.06 | 0.94 |
| 2 | 2 | 21.7 | 12.7 | 1.043 | -5.48 | 1.06 | 0.94 |
| 3 | 1 | 2.4 | 1.2 | 1.021 | -7.96 | 1.06 | 0.94 |
| 4 | 1 | 7.6 | 1.6 | 1.012 | -9.62 | 1.06 | 0.94 |
| 5 | 2 | 94.2 | 19 | 1.01 | -14.37 | 1.06 | 0.94 |
| 6 | 1 | 0 | 0 | 1.01 | -11.34 | 1.06 | 0.94 |
| 7 | 1 | 22.8 | 10.9 | 1.002 | -13.12 | 1.06 | 0.94 |
| 8 | 2 | 30 | 30 | 1.01 | -12.1 | 1.06 | 0.94 |
| 9 | 1 | 0 | 0 | 1.051 | -14.38 | 1.06 | 0.94 |
| 10 | 1 | 5.8 | 2 | 1.045 | -15.97 | 1.06 | 0.94 |
| 11 | 2 | 0 | 0 | 1.082 | -14.39 | 1.06 | 0.94 |
| 12 | 1 | 11.2 | 7.5 | 1.057 | -15.24 | 1.06 | 0.94 |
| 13 | 2 | 0 | 0 | 1.071 | -15.24 | 1.06 | 0.94 |
| 14 | 1 | 6.2 | 1.6 | 1.042 | -16.13 | 1.06 | 0.94 |
| 15 | 1 | 8.2 | 2.5 | 1.038 | -16.22 | 1.06 | 0.94 |
| 16 | 1 | 3.5 | 1.8 | 1.045 | -15.83 | 1.06 | 0.94 |
| 17 | 1 | 9 | 5.8 | 1.04 | -16.14 | 1.06 | 0.94 |
| 18 | 1 | 3.2 | 0.9 | 1.028 | -16.82 | 1.06 | 0.94 |
| 19 | 1 | 9.5 | 3.4 | 1.026 | -17 | 1.06 | 0.94 |
| 20 | 1 | 2.2 | 0.7 | 1.03 | -16.8 | 1.06 | 0.94 |
| 21 | 1 | 17.5 | 11.2 | 1.033 | -16.42 | 1.06 | 0.94 |
| 22 | 1 | 0 | 0 | 1.033 | -16.41 | 1.06 | 0.94 |
| 23 | 1 | 3.2 | 1.6 | 1.027 | -16.61 | 1.06 | 0.94 |
| 24 | 1 | 8.7 | 6.7 | 1.021 | -16.78 | 1.06 | 0.94 |
| 25 | 1 | 0 | 0 | 1.017 | -16.35 | 1.06 | 0.94 |
| 26 | 1 | 3.5 | 2.3 | 1 | -16.77 | 1.06 | 0.94 |
| 27 | 1 | 0 | 0 | 1.023 | -15.82 | 1.06 | 0.94 |
| 28 | 1 | 0 | 0 | 1.007 | -11.97 | 1.06 | 0.94 |
| 29 | 1 | 2.4 | 0.9 | 1.003 | -17.06 | 1.06 | 0.94 |
| 30 | 1 | 10.6 | 1.9 | 0.992 | -17.94 | 1.06 | 0.94 |

Table A2: Generator Data

| Bus | Pg | Qg | Qm | Qm | Vg | mB | Pma | Pmi |
|-----|-------|-------|----|-----|-------|-----|-------|-----|
| No | | | ax | in | | ase | х | n |
| 1 | 260.2 | -16.1 | 10 | 0 | 1.06 | 100 | 360.2 | 0 |
| 2 | 40 | 50 | 50 | -40 | 1.045 | 100 | 140 | 0 |
| 5 | 0 | 37 | 40 | -40 | 1.01 | 100 | 100 | 0 |
| 8 | 0 | 37.3 | 40 | -10 | 1.01 | 100 | 100 | 0 |
| 11 | 0 | 16.2 | 24 | -6 | 1.082 | 100 | 100 | 0 |
| 13 | 0 | 10.6 | 24 | -6 | 1.071 | 100 | 100 | 0 |

Table A3: Line Data

| Line | From | То | r | Х | b | ratio |
|------|------|-----|--------|--------|--------|-------|
| No | bus | bus | | | | |
| 1 | 1 | 2 | 0.0192 | 0.0575 | 0.0528 | 0 |
| 2 | 1 | 3 | 0.0452 | 0.1652 | 0.0408 | 0 |
| 3 | 2 | 4 | 0.057 | 0.1737 | 0.0368 | 0 |
| 4 | 3 | 4 | 0.0132 | 0.0379 | 0.0084 | 0 |
| 5 | 2 | 5 | 0.0472 | 0.1983 | 0.0418 | 0 |
| 6 | 2 | 6 | 0.0581 | 0.1763 | 0.0374 | 0 |
| 7 | 4 | 6 | 0.0119 | 0.0414 | 0.009 | 0 |
| 8 | 5 | 7 | 0.046 | 0.116 | 0.0204 | 0 |
| 9 | 6 | 7 | 0.0267 | 0.082 | 0.017 | 0 |
| 10 | 6 | 8 | 0.012 | 0.042 | 0.009 | 0 |
| 11 | 6 | 9 | 0 | 0.208 | 0 | 0.978 |
| 12 | 6 | 10 | 0 | 0.556 | 0 | 0.969 |
| 13 | 9 | 11 | 0 | 0.208 | 0 | 0 |
| 14 | 9 | 10 | 0 | 0.11 | 0 | 0 |
| 15 | 4 | 12 | 0 | 0.256 | 0 | 0.932 |
| 16 | 12 | 13 | 0 | 0.14 | 0 | 0 |
| 17 | 12 | 14 | 0.1231 | 0.2559 | 0 | 0 |
| 18 | 12 | 15 | 0.0662 | 0.1304 | 0 | 0 |
| 19 | 12 | 16 | 0.0945 | 0.1987 | 0 | 0 |
| 20 | 14 | 15 | 0.221 | 0.1997 | 0 | 0 |
| 21 | 16 | 17 | 0.0524 | 0.1923 | 0 | 0 |
| 22 | 15 | 18 | 0.1073 | 0.2185 | 0 | 0 |
| 23 | 18 | 19 | 0.0639 | 0.1292 | 0 | 0 |
| 24 | 19 | 20 | 0.034 | 0.068 | 0 | 0 |
| 25 | 10 | 20 | 0.0936 | 0.209 | 0 | 0 |
| 26 | 10 | 17 | 0.0324 | 0.0845 | 0 | 0 |
| 27 | 10 | 21 | 0.0348 | 0.0749 | 0 | 0 |
| 28 | 10 | 22 | 0.0727 | 0.1499 | 0 | 0 |
| 29 | 21 | 22 | 0.0116 | 0.0236 | 0 | 0 |
| 30 | 15 | 23 | 0.1 | 0.202 | 0 | 0 |
| 31 | 22 | 24 | 0.115 | 0.179 | 0 | 0 |
| 32 | 23 | 24 | 0.132 | 0.27 | 0 | 0 |
| 33 | 24 | 25 | 0.1885 | 0.3292 | 0 | 0 |
| 34 | 25 | 26 | 0.2544 | 0.38 | 0 | 0 |
| 35 | 25 | 27 | 0.1093 | 0.2087 | 0 | 0 |
| 36 | 28 | 27 | 0 | 0.396 | 0 | 0.968 |
| 37 | 27 | 29 | 0.2198 | 0.4153 | 0 | 0 |
| 38 | 27 | 30 | 0.3202 | 0.6027 | 0 | 0 |
| 39 | 29 | 30 | 0.2399 | 0.4533 | 0 | 0 |
| 40 | 8 | 28 | 0.0636 | 0.2 | 0.0428 | 0 |
| 41 | 6 | 28 | 0.0169 | 0.0599 | 0.013 | 0 |