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Performance Evaluation of Direction of Arrival Estimation using Uniform and Non-uniform Linear Arrays

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Abstract In sensor array signal processing, the direction of arrival (DOA) estimation denotes the angle at which an electromagnetic or acoustic wave arrives at an array of antennas or sensors. Using an array of antennas has an advantage over the single antenna in achieving an improved performance by applying Multiple Signal Classification (MUSIC) algorithm. This paper focuses on estimating the DOA using uniform linear array (ULA) and non-uniform linear array (NLA) of antennas to analyze the performance factors that affect the accuracy and resolution of the system based on MUSIC algorithm. The direction of arrival estimation is simulated on a MATLAB platform with a set of input parameters such as array elements, signal to noise ratio, number of snapshots and number of signal sources. An extensive simulation has been conducted and the results show that the NLA with DOA estimation for co-prime array can achieve an accurate and efficient DOA estimation.

Keywords Direction of Arrival (DOA) estimation, Multiple Signal Classification (MUSIC), Non-Uniform Linear Array (NLA), Uniform Linear Array (ULA).

1. Introduction

In the past decades, many array models such as the uniform linear array (ULA), uniform circular array, uniform rectangular array, and non-uniform linear array (NLA) have been utilized towards the achievement of the direction of arrival (DOA) estimation [1]. To get an accurate DOA estimation of the narrowband signal sent from a far field source to a receiving antenna array can increase the wireless communication systems capacity [2]. Therefore, putting effort on improving DOA estimation methods is a key to developing the quality of wireless networks.

In many practical signal processing problems, the received narrowband signal depends on a set of constant parameters such as number of array elements, number of snapshots, element spacing, angular separation, signal-to-noise ratio. The objective of this paper is to estimate those parameters for the effectiveness of the systems. A vast number of methods have been employed to solve such problems including the Maximum Likelihood (ML), Burg's Maximum Entropy (ME), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPIRIT), Multiple Signal Classification (MUSIC), Smooth-MUSIC and Root-MUSIC methods [3], [4]. The Multiple Signal Classification is the most popular method that works for any array shapes. This method is suitable especially when there are a limited number of sensors [5]. This algorithm is based on exploiting the eigenstructure of the input covariance matrix. The direction of arrival of multiple signal sources can be easily estimated by identifying the peaks of a MUSIC spatial spectrum [6].

Most of the methods used to locate the DOA consider the spacing distance between two adjacent array elements to be a half wavelength on ULA. However in wireless communication, there are some cases where such half wavelength minimum spacing is not applicable; for instance many parabola antennas, their physical size are designed to have a large size for enhanced directivity. Also, in an array that operates over a wide spectrum; for example, over-the-horizon radar (OTHR) is a unique radar system that performs wide area surveillance by exploiting the reflective and refractive nature of high-frequency radio wave propagation through the ionosphere [7].

Recently, a non-uniform linear array in a form of co-prime array has been proposed [8]. Its most remarkable property is that it increases the degrees of freedom. In addition, the autocorrelation of signals can be estimated in a much denser spacing other than the physically sparse sampling spacing, and sinusoids in noise can be estimated in a more effective way. Due to the useful properties of the NLA, its importance has been realized and has been the object of research in the last few years. Some researchers such as Pal and Vaidyanathan in [9], [10] proposed a new method for a super resolution spectral estimation from the perspective of degree of freedom increase. Further to these efforts, Weng and Petar in [11] proposed a search-free DOA algorithm for co-prime arrays by using a projection-like method to eliminate the phase ambiguities for obtaining the unique estimation of DOA.

In this paper, a comparative study of DOA estimation using uniform and non-uniform linear array antennas is proposed. It underlies the factors that affect the accuracy and resolution of the system based on multiple signal classification (MUSIC) algorithm. The performance evaluation of the MUSIC algorithm under ULA and NLA is given.

The remainder of this paper is organized as follows. In Section 2, the signal model of the uniform linear array is presented. In Section 3, the description of a detailed MUSIC algorithm implementation is given. Section 4, shows a non-uniform linear array with combined MUSIC algorithm. Simulation and discussion of the results are presented in Section 5. Finally, conclusion and recommendation for future work is highlighted in Section 6.

2. Signal Model of Uniform Linear Array

Consider a linear array antenna with M antenna elements that are equally spaced with a distance d that is strictly equal to a half wavelength ($\boldsymbol{d} = \frac{\lambda}{2}$). Assume that there are narrowband signal sources ($\boldsymbol{K}$) located at $\boldsymbol{\theta_1}, \boldsymbol{\theta_2}, \dots, \boldsymbol{\theta_K}$ with signal powers $\boldsymbol{\sigma_1^2}, \boldsymbol{\sigma_2^2}, \dots, \boldsymbol{\sigma_K^2}$. The incident signals and the noise are uncorrelated. In addition the incident signals are themselves uncorrelated. Let the number of signal sources be less than the number of antenna elements $(\boldsymbol{K} < \boldsymbol{M})$. The steering vector for $\boldsymbol{k^{th}}$ source located at $\boldsymbol{\theta_k}$ is with $\boldsymbol{k^{th}}$ element $\boldsymbol{e^{j(2\pi/\lambda)d_l \sin(\theta_k)}}$, where $\boldsymbol{d_l}$ is the antennas location and $\boldsymbol{\lambda}$ is the wavelength as shown in Fig. 1.

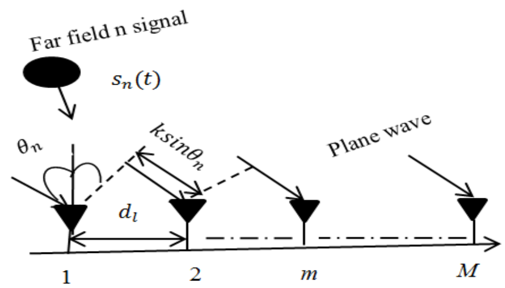

Fig. 1. A plane wave incident on a uniform linear array antenna

The signal collected by all antennas at time (t) can be expressed as:

$$\boldsymbol{X(t) = As(t) + n(t)} \quad (1)$$

Where $X(t) = [x_1(t)\ x_2(t) \dots x_M(t)]^T$ is a vector received by the array antenna,
$A = [a(\theta_1)\ a(\theta_2) \dots a(\theta_K)]$ is a steering vector. Its expression is shown in (2), $s(t)$ is the signal vector generated by the source and $n(t) = [n_1(t)\ n_2(t)\ \dots\ n_M(t)\ ]^T$ is the additive white Gaussian noise.

$$\boldsymbol{a(\theta_k)} = \begin{bmatrix} \boldsymbol{1} & \boldsymbol{1} & \dots & \boldsymbol{1} \\ \boldsymbol{e^{-j\varphi_1}} & \boldsymbol{e^{-j\varphi_2}} & \dots & \boldsymbol{e^{-j\varphi_K}} \\ \dots & \dots & \dots & \dots \\ \boldsymbol{e^{-j\varphi_1(M-1)}} & \boldsymbol{e^{-j\varphi_2(M-1)}} & \dots & \boldsymbol{e^{-j\varphi_K(M-1)}} \end{bmatrix} \quad (2)$$

where $\boldsymbol{\varphi_k = \frac{2\pi d}{\lambda} sin\theta_k}$

The narrowband signal from (1) can be expressed as:

$$\boldsymbol{S_k(t) = s_k(t)\exp\{j\omega_k(t)\}}, \tag{3}$$

30 Where $s_k(t)$ is the complex envelope of $S_k(t)$ and $\omega_k(t)$ is the angular frequency of $S_k(t)$.
As assumed all signals have the same center frequency, therefore:

$$\boldsymbol{\omega_k = \omega_0 = \frac{2\pi c}{\lambda}} \tag{4}$$

where c is the speed of the light and λ is the wavelength of the signal.

According to the narrowband assumption, the following approximation is valid:

$$\boldsymbol{S_k(t - t_1) \approx s_k(t)} \tag{5}$$

The delayed wavefront signal is:

$$\boldsymbol{S_k(t - t_1) = s_k(t)\exp[j\omega_0(t - t_1)] = s_k(t)\exp[j\omega_0(t - t_1)]} \tag{6}$$

The output signal of the *m*th element is:

$$X_m(t) = \\ = \sum_{k=1}^{K} s_k(t)\exp\left[-j(m-1)\frac{2\pi d\sin\theta_k}{\lambda}\right] + n_m(t) \quad (7)$$

From (6) the output steering array vector is:

$$a_m(t) = \exp\left[-j(m-1)\frac{2\pi d\sin\theta_k}{\lambda}\right] \tag{8}$$

3. MUSIC Algorithm Implementation

Multiple signal classification is a subspace technique based on exploiting the eigenstructure of input covariance matrix [12]. Through a set of input parameters such as number of array elements, number of snapshots, element spacing, angular separation, signal-to-noise ratio and MUSIC algorithm, the DOA is estimated with high resolution. This leads to high quality of wireless communication.

The autocorrelation of the received signal is:

$$R_x = E[XX^H] \tag{9}$$

Where X^H is the conjugate transpose matrix of X. The noise is assumed to be zero mean and additive white Gaussian and is uncorrelated to the signal.

Replacing (1) into (9), the covariance matrix is obtained: 31

$$\begin{aligned} R_x &= E[(As + N)(As + N)^H] \\ &= AE[ss^H]A^H + E[NN^H] \\ &= AR_SA^H + R_N \end{aligned} \tag{10}$$

Where $R_S = E[ss^H]$ is called source signal correlation matrix, $R_N = \sigma^2 I$ is a noise correlation matrix.
If $(\lambda_1, \lambda_2, \ldots .\lambda_M)$ are eigenvalues of spatial correlation matrix R_x, then the relationship between eigenvalue and the correlation matrix is given as:

$$R_x - \lambda_i \mathrm{I} = 0 \tag{11}$$
$$AR_SA^H + \sigma^2 I - \lambda_i \mathrm{I} = 0 \tag{12}$$
$$AR_SA^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{13}$$

Therefore, eigenvectors v_i of AR_SA^H associated with a certain eigenvalue are obtained using (14);

$$v_i = \sigma^2 - \lambda_i \tag{14}$$

The eigenvalues are sorted according to their magnitudes $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M \geq 0$. The larger eigenvalues correspond to signal subspace, while small eigenvalues corresponds to noise. The largest eigenvalues are all values greater than K that is $(M - K)$, and the rest are all values that are less than K. Thus, MUSIC "Spatial spectrum" in [12] is defined as:

$$P_{music}(\theta) = \frac{1}{a^H(\theta)E_N E_N{}^H a(\theta)} \tag{15}$$

The multiple signal classification (MUSIC) algorithm can be summarized as follow:
Step 1: The data is collected to form the correlation matrix R_x.
Step 2: The eigenstructure of the covariance matrix R_x is decomposed.
Step 3: Let the number of signal sources be K.
Step 4: N columns are chosen to form the noise subspace E_N.
Step 5: P_{music} versus θ is evaluated.
Step 6: The spectrum function is determined; then an estimation of DOA is obtained by the peak-searching *k*.

4. Non-Uniform Linear Array with Combined MUSIC Algorithm

A system with the array antennas linearly spaced, the different array elements spacing and the signal phase differences is considered. In this case, one non-uniform linear array can be decomposed into two uniform linear arrays as shown in Fig. 2 and Fig. 3 respectively. The direction of arrival is estimated by combining MUSIC results from the two co-prime arrays.

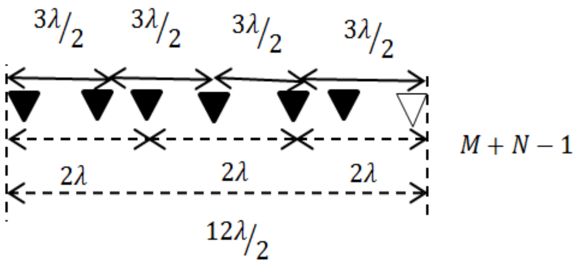

Fig. 2. Co-prime non-uniform linear array

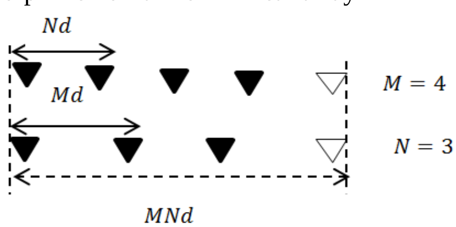

Fig. 3. Two co-prime uniform linear arrays

The received signal vector of each sub-array at the *t-th* time slot can be defined as:

$$X_M(t) = A_M s(t) + n(t) \tag{16}$$
$$X_N(t) = A_N s(t) + n(t) \tag{17}$$

The array output vector of the co-prime array is given by:

$$Y(t) = A(t)s(t) + n(t), \tag{18}$$

According to the far-field assumption, the steering vector corresponding to the k^{th} source is:

$$a_k = \\ = \left[1, e^{-j\frac{2\pi}{\lambda}d_1 \sin(\theta_k)}, \dots, e^{-j\frac{2\pi}{\lambda}d_i \sin(\theta_k)}, e^{-j\frac{2\pi}{\lambda}d_{(M+N-1)} \sin(\theta_k)}\right]^T \tag{19}$$

Where $d_l(l = 1,2, \dots, M + N - 1)$

Steering arrays of each linear array are given by:

$$a_{Mk} = \\ = \left[1, e^{-j\pi N sin(\theta_k)}, \dots, e^{-j\pi(M-1)N sin(\theta_k)}\right]^T \tag{20}$$

$$a_{Nk} = \\ = [1, e^{-j\pi M sin(\theta_k)}, \dots, e^{-j\pi(N-1)M sin(\theta_k)}]^T \tag{21}$$

By obtaining each sample of covariance matrices from (16) and (17), the two decomposed uniform linear sub-arrays give:

$$R_{xM} = E[X_M(t)X_M(t)^H] \tag{22}$$

$$R_{xN} = E[X_N(t)X_N(t)^H] \tag{23}$$

Replacing the received signal vector of each sub-array (16) and (17) by their values, the following is obtained:

$$R_{xM} = E[(A_M s(t) + n(t))(A_M s(t) + n(t))^H] \\ = A_M E(s(t)s(t)^H) A_M{}^H + E[NN^H] \\ = A_M R_{SM} A_M{}^H + R_N \tag{24}$$

$$R_{xN} = E[(A_N s(t) + n(t))(A_N s(t) + n(t))^H] \\ = A_N E(s(t)s(t)^H) A_N{}^H + E[NN^H] \\ = A_N R_{SN} A_N{}^H + R_N \tag{25}$$

Where $R_{SM} = E[s(t)s(t)^H]$ and $R_{SN} = E[s(t)s(t)^H]$ are source signal correlation matrices of subarrays M and N respectively, $R_N = \sigma^2 I$ is a noise correlation matrix for each subarray.

If $(\lambda_1, \lambda_2, \dots .\lambda_M)$ and $(\lambda_1, \lambda_2, \dots .\lambda_N)$ are eigenvalues of the spatial correlation matrix R_{xM} and R_{xN}, then the performance of eigenvalue associated with a particular eigenvector for each sub-array is given as:

$$R_x - \lambda_i \mathrm{I} = 0 \tag{26}$$

$$A_M R_{SM} A_M{}^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{27}$$

$$A_N R_{SN} A_N{}^H + (\sigma^2 - \lambda_i)\mathrm{I} = 0 \tag{28}$$

The eigenvectors v_i of $A_M R_{SM} A_M{}^H$ and $A_N R_{SN} A_N{}^H$ from (24) and (25) are obtained using (28) whereby v_i is:

$$v_i = \sigma^2 - \lambda_i \tag{29}$$

Therefore applying eigen-decomposition to the sample covariance matrices in (24) and (25) yields:

$$R_{xM} = E_{sM} \Lambda_{sM} E_{sM}{}^H + E_{nM} \Lambda_{nM} E_{nM}{}^H \tag{30}$$

$$R_{xN} = E_{sN} \Lambda_{sN} E_{sN}{}^H + E_{nN} \Lambda_{nN} E_{nN}{}^H \tag{31}$$

Then, according to the orthogonality between the signal subspace and the noise subspace of each sub-array, the

MUSIC spatial pseudo-spectrum of the two decomposed linear subarrays M and N respectively, are:

$$P_{MUSIC_M}(\theta) = \frac{1}{a_M(\theta)^H E_{nM} E_{nM}{}^H a_M(\theta)} \tag{32}$$

$$P_{MUSIC_N}(\theta) = \frac{1}{a_N(\theta)^H E_{nN} E_{nN}{}^H a_N(\theta)} \tag{33}$$

5. Simulation Results and Discussion

5.1. Introduction

The simulation of MUSIC algorithm is carried out on a MATLAB platform. This section is subdivided into four parts.

Firstly, the simulation results experiment 1 are validated using published data in the cited literature. Secondly in experiment 2, a uniform linear array (ULA) of 13 elements ($M = 13$) is assumed. The same frequency used in wireless communication (Wi-Fi) which is 2.4 GHz (Freq. = 2.4GHz) is considered, with 200 snapshots ($N = 200$), SNR = -5dB and the two uncorrelated estimated signals arriving at angular direction of 60^0 and 68^0, ($DOA = [60° \; 68°]$) at ULA are used.

Thirdly in experiment 3, the simulation with two superimposed uniform linear arrays with the same inputs parameters as in the experiment 2 are assumed. Lastly in experiment 4, a simulation result of a non-uniform linear array (NLA) is given. The same input parameters such as number of array element, signal to noise ratio, number of snapshots, and number of arriving signal and at the same frequency are considered.

For experiments 2 and 3, the distance between two adjacent antenna elements is assumed to be $d = \frac{\lambda}{2}$; then the fourth experiment, the distance greater than a half wavelength $d > \frac{\lambda}{2}$ is assumed. In all cases, the Gaussian noise is assumed to be zero mean and white and is uncorrelated to the signal. Also, the direction of arrival θ_k is assumed to be within the angle interval $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Here the idea is to vary signal to noise ratio (SNR) and number of array element to analyze the performance results of the three experiments and then check on the factors that affect the accuracy. Finally, compare both ULA and NLA in terms of the resolution and accuracy of the system. This is achieved by comparing the Mean Square Error (MSE) of the direction of arrival estimation for a ULA and NLA performances through MUSIC algorithm.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(\theta - \hat{\theta}_i)^2$$

With $i = 1,2, \ldots, n$, where n is the number of trials, θ is the true angle of arrival and $\hat{\theta}_i$ is an i^{th} estimated angle of arrival.

5.2. Experiment 1: Validation of results

The simulation results were justified by using existing data in the literature. The dashed lines in Fig. 4 are results from [12] while the continuous lines are the present work with the increased signal to noise ratio. The Fig. 4 gives a performance of MUSIC spectrum for six different variation of the signal to noise ratio. While keeping the other input parameters such as array size: $M = 20$ elements, number of snapshots: $N = 200$ snapshots, true DOA: $10°, 30°$ *and* $50°$ and the signal frequency: $Freq = 2.4GHz$. In Dhering's work, the signal to noise ratio was varied from $15dB, 25dB$ *to* $35dB$ and for the current work, the signal to noise ratio is varied from $20dB, 30dB$ *to* $40dB$. As observed, there is a good agreement between the simulated results and the published data in [12].

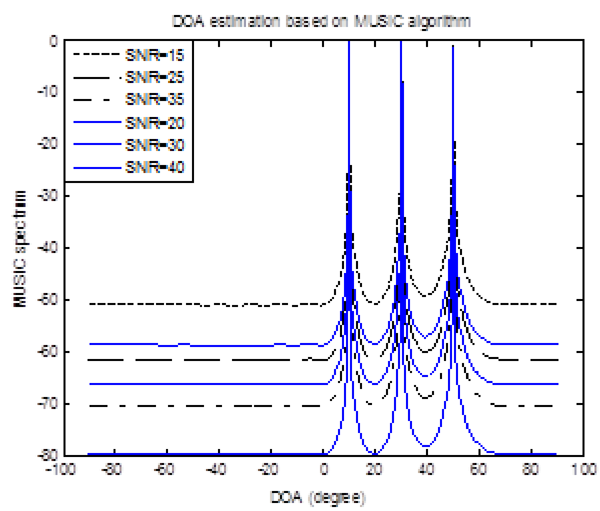

Fig. 4. MUSIC spectrum for SNR variation

5.3. Experiment 2: DOA estimation with ULA

In this experiment, the results were obtained using one uniform linear array based on MUSIC algorithm for the input parameter set: $\theta_1 = 60^{\circ}, \theta_2 = 68^{\circ}, SNR = -5dB, M = 13, N = 100, Freq = 2.4GHz, k = 2, d = \frac{\lambda}{2}$. The results are given in Fig. 5.

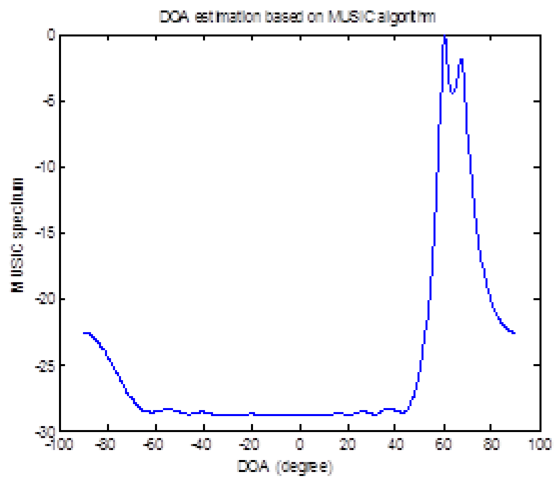

Fig. 5. DOA estimation based on MUSIC algorithm for one ULA

5.4. Experiment 3: DOA estimation with two superimposed ULA

In this experiment, the results were obtained using two uniform linear arrays based on MUSIC algorithm for the input parameter set: $\theta_1 = 60^{\circ}$,$\theta_2 = 68^{\circ}$,$SNR = -5dB$, $L = 9$,$J = 5$, $N = 100$, $Freq = 2.4GHz$, $k = 2$, $d = \frac{\lambda}{2}$. The results are given in Fig. 6.

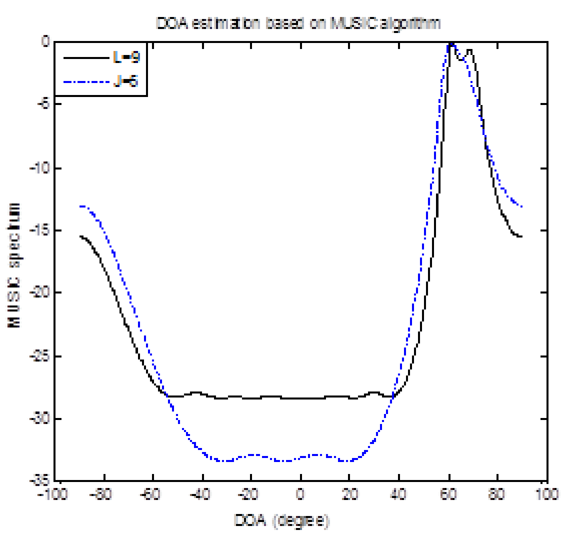

Fig. 6. DOA estimation based on MUSIC algorithm for two ULA

5.5 Experiment 4: DOA estimation with two superimposed ULA

In this experiment, the simulation results of a non-uniform linear array were obtained for the input parameter set: $\theta_1 = 60^{\circ}$, $\theta_2 = 68^{\circ}$,$SNR = -5dB$, $L = 9$,$J = 5$, $N = 100$,$Freq = 2.4GHz$, $k = 2$, $d > \frac{\lambda}{2}$. The results are given in Fig.7.

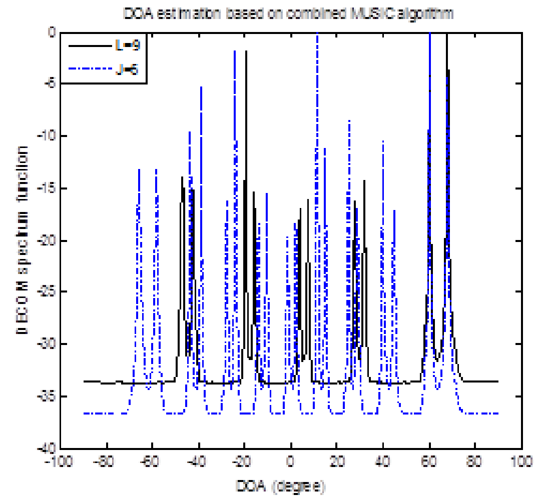

Fig. 7. DOA estimation based on combined MUSIC algorithm for non-uniform linear array

5.6 Discussion

The multiple signal classification method gives a high resolution and is accurate for DOA estimation especially when the number of array elements, the signal to noise ratio and the number of snapshots are high. However, MUSIC strength varies from one array arrangement to another as indicated in Table 1. The simulations experiments were carried out with 13 array elements for one ULA as indicated in Fig. 5, two superimposed uniform linear arrays of 9 elements and 5 elements as indicated in Fig. 6 and DECOM (9,5) for two decomposed uniform linear sub-arrays which form a non-uniform linear array as shown in Fig. 7. The DOA at 60^0 and 68^0 in the angle interval $(-\frac{\pi}{2},\frac{\pi}{2})$ are assumed. Therefore, the observed direction of arrival estimation is shown in Table 1.

Table 1: variation of DOA estimation via different array arrangement

DOA	Array elements			
	13	9 and 5		(9,5)
		9	5	
60^0	60.5^0	61^0	62^0	60^0
68^0	67.5^0	66.5^0	62^0	68^0

The results from Fig. 5, Fig. 6 and Fig. 7 are summarized in Table 1. Due to the Gaussian white noise environment, the experiments has been repeated 8 times to be able to analyze errors using MSE. The true DOA of arrivals are designed as θ_1 and θ_2 and the estimated DOA of arrivals are designed as $\hat{\theta}_1$ and $\hat{\theta}_2$.

It can be noted that from experiment 2, using one ULA of 13 elements ($M = 13$), with the true angles of arrival ($\theta_1 = 60^\circ, \theta_2 = 68^\circ$), the estimated direction of arrivals are 60.5° and 67.5° ($\hat{\theta}_1 = 60.5^\circ, \hat{\theta}_2 = 67.5^\circ$) for one trial. The analysis of error for 8 trials, the Mean Square Error: $MSE_1 = 0.0794$ and $MSE_2 = 0.004050$ for the arrivals at $\theta_1 = 60^\circ$ and $\theta_2 = 68^\circ$ respectively.
Moreover in experiment 3, the use of 9 elements and 5 elements $[L = 9, J = 5]$ for superimposed ULAs is used with the angle of arrival is $\theta_1 = 60^\circ, \theta_2 = 68^\circ$. Thus, the observe direction of arrivals estimation from L are 61° and 66.5° ($\hat{\theta}_1 = 61°, \widehat{\theta}_2 = 66.5°$), and from J are 62° and 62° ($\hat{\theta}_1' = 62°, \ \hat{\theta}_2{}' = 62°$) for one trial. The analysis of error for 8 trials, the Mean Square Error for 5 uniform array elements: $MSE_1 = 8.4429$ and $MSE_2 = 8.0755$ and for 9 uniform array elements: $MSE_1{}' = 0.2246$ and $MSE_2{}' = 0.2159$ for the arrivals at $\theta_1 = 60^\circ$ and $\theta_2 = 68^\circ$ respectively.

Furthermore in experiment 4, the use DECOM (9, 5) that is $[L = 9, J = 5]$ in fig. 7. When applying the same characteristics as in the second experiment with the same true value of the angle of arrivals are $\theta_1 = 60^\circ, \theta_2 = 68^\circ$, the observed direction of arrivals estimation from L are 60° and 68° ($\hat{\theta}_1 = 60°, \widehat{\theta}_2 = 68°$), then from J are 60° and 68° ($\hat{\theta}_1{}' = 60°, \widehat{\theta}_2{}' = 68°$) for one trial. The analysis of error for 8 trials, the Mean Square Error for 9 uniform sub-array elements: $MSE_1 = 0.00000076$ and $MSE_2 = 0.00000731$ and for 5 uniform array elements: $MSE_1{}' = 0.0000000738$ and $MSE_2{}' = 0.0000000563$ for the arrivals at $\theta_1 = 60^\circ$ and $\theta_2 = 68^\circ$ respectively.

Thus, the performance of a non-uniform linear array of a decomposed ULA for the corresponding co-prime array with combined MUSIC algorithm has a good estimate DOA according to the Mean Square Error. This demonstrates the better performance regarding the high resolution and accurate of the non-uniform linear array (NLA) which contribute in improving the wireless communication capacity.

6. Simulation Results and Discussion

In this paper, a comparison method of estimating DOA using uniform and non-uniform linear array antennas by underlying the factors that affect the accuracy and resolution was given. The simulation of the variation of direction of arrival estimation via different array arrangements based on Mean Square Error showed that non-uniform linear array has better estimate which result in increasing the wireless capacity. A mathematical model of the received signal and the detailed MUSIC algorithm when using ULA and NLA for the direction of arrival has been given.

Through intense simulations in MATLAB, the variation of the inputs parameter sets have shown that the more the increase SNR, array elements, snapshots the more accurate and high resolution is the DOA estimation. Moreover, it is shown that non-uniform linear array based on DOA estimation of co-prime array by combining the MUSIC results of the corresponding two decomposed uniform linear arrays is the best array arrangement due to its increase of the degree of freedom and it is the more accurate and high resolution for the DOA estimation compared to ULA. As observed in the simulation results, the non-uniform linear array helps to get a good estimate of DOA which leads to an improvement of the wireless communication systems capacity.

In the future, the application of the two dimensions of NLA in form of DOA estimation for co-prime array in the real time environments need to be considered.

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